THE INFORMATIONAL CONTENT OF INTEREST RATES: FORECASTABILITY AND SECRECY IN CENTRAL BANKING

By

Massimo Gesua' sive Salvadori

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Supervisor:	Christopher Bliss
Internal Examiner:	Nicholas Dimsdale
External Examiner:	Marcus Miller

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Author: **Massimo Gesua' sive Salvadori**

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Signature of Author

To Lea and Lorenzo

for they are the future.

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Abstract

We investigate three questions related to the economics of information of monetary policy: i) Should the voting records of individual members of the Interest Rates Setting Panel of a Monetary Union be divulged to the public?; ii) Is the observed pattern of interest rate smoothing, the partial adjustment mechanism for nominal interest rates and the low ratio of reversals to total changes in the setting of interest rates justified or, instead, does it imply that the response by Central Banks to news is invariably an overly timid one? iii) What are the implications of information secrecy if we follow Romer et al. (Romer and Romer 2000) in assuming that the Central Bank is endowed with asymmetric and superior information as to the path of macroeconomic fundamentals?

We study in Chapter 2 the problem of voting transparency in a Monetary Union and accept at face value the ECB's claim that transparent voting induces partisan behavior by policy-makers. We set the analysis in a simple economic geography framework. If the issue of industry location is held exogenous to monetary policy, we find that voting secrecy is welfare optimal. We construct a simple general equilibrium framework to show that the welfare comparison between voting transparency and voting secrecy is, instead, ambiguous when the choice of industrial location is modeled to be endogenous to monetary policy.

We find in Chapter 3 that the assumption that the Central Bank is endowed with asymmetric and superior information as to the path of macroeconomic fundamentals imparts some smoothness to interest rates. We also show that the choice of information transparency over information secrecy and the mandate that the Central Bank should publish detailed minutes of its meetings imply that interest rates are less likely to stay on hold and more likely to move by a large magnitude. We find that the welfare comparison between information secrecy and transparency is ambiguous and state conditions under which one is welfare superior to the other. We formulate a conjecture that our model is consistent with a high continuations to total changes ratio and we find some results analogous to limit pricing behavior (Milgrom and Roberts 1982).

We construct in Chapter 4 a learning model of the yield curve and interpret the credibility of monetary policy as being represented by the Central Bank's capability to affect a large movement in the medium and long portion of the yield curve with a relatively small change in the current short-run interest rate. We find that a positive pattern of historical serial correlation in interest rate changes implies that the Central Bank can *bring into effect a large movement in the long portion of the yield curve with a small change in short-run rates so that interest rate smoothing does not necessarily imply an excessively timid response by policy-makers to macroeconomic shocks. We also find that the short-term rate is increasing in its lag and in its lagged rate of change so that monetary policy exhibits a partial adjustment mechanism and a short-run path dependent behavior.*

KEYWORDS: VOTING TRANSPARENCY, INFORMATION TRANSPARENCY, INTEREST RATES SMOOTHING.

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Chapter 1

Overview

1.1 The Theme of the Thesis

We investigate in this thesis three questions related to the economics of information of monetary policy: i) Should the voting records of individual members of the Interest Rate Setting Panel of a Monetary Union be divulged to the public? Or, rather, can we hold the current ECB's policy of not divulging such records for seventeen years to be welfare enhancing as it is claimed by its architects (Issing 1999)?

ii) Is the pattern of interest rate smoothing, the partial adjustment mechanism for nominal interest rates and the low ratio of reversals to total changes in interest rates* documented by the literature (see, *inter alia*, Goodhart (Goodhart 1997), Clarida et al. (Clarida, Gali, and Gertler 1999) and Sack et al. (Sack and Wieland 2000)) an indication that Central Banks act *too little and too late* in counter-acting news on macroeconomic fundamentals as argued by a number of authors (such as, for instance, Ball (Ball 1999), Goodhart (Goodhart 1997) and Goodfriend (Goodfriend 1991))? Or, instead, can we account for such behavior by deriving some findings that are at least suggestive of the fact that the smoothness in short-term rates can be justified and does not necessarily

*The ratio of reversals to total changes is constructed as follows. Let the term *reversal* indicate a change in the value of the given instrument for monetary policy, typically a measure of the one-month repo rate, of a different sign to the last one the Central Bank has implemented; let the term continuation, instead, denote a change in the value for the instrument of monetary policy of the same sign as the last innovation announced by the Central Bank. If the term total changes represents the sum of continuations and reversals, the reversals to total changes ratio can be employed as a measure of the frequency with which the Central Bank inverts the direction of interest rates changes.

imply that the response by the Central Bank to news is invariably an overly timid one?

iii) What are the implications of the regime of information secrecy adopted by some Central Banks (an high example of which being the FED's practice of revealing with a lag of no less that five years both the macroeconomic forecasts by its staff and by members of the FOMC, a procedure which some agents have tried to terminate in the eighties by bringing the FED to court with an unsuccessful action (Goodfriend 1986))? And is this behavior endowed with any welfare rising consequence so that it can be somewhat rationalized? Moreover, is the practice of information secrecy followed by some Central Banks best understood if we follow Romer et al. (Romer and Romer 2000) in assuming that the Central Bank is endowed with asymmetric and superior information as to the path of macroeconomic fundamentals in an horizon of up to two years of length? And, on a related note, can we account for the smoothness in the path of interest rates as being the result of the Central Bank's attempt not to reveal information that might induce agents to revise their planned paths for consumption and investment in a pro-cyclical manner, while financial markets might be destabilized by large and sudden movements in interest rates as the ECB's Chairman Duisenberg was recently widely quoted in the press as firmly stating ((Duisenberg 2001), p.12)?

Each of the three central chapters of the thesis tries to address one of these three questions. Chapter 2 investigates the issue of voting transparency in a Monetary Union, as detailed in point i); Chapter 3, instead, studies the issue of informational secrecy relating to question iii) but also touches upon the interest rate smoothing problem of question ii); Chapter 4, finally, analyzes a possible rationale behind the practice of the interest rate smoothing procedure as described by research question ii).

It must be admitted that the three central chapters are only weakly inter-related, perhaps with the exception of Chapter 3 and Chapter 4, which both have a bearing on the analysis of interest rate smoothing and partial adjustment rules for monetary policy. However, we would like to suggest a possible methodological pattern of unity among our three central areas of investigation.

1.1.1 A Common Sequence

At one level, the three Central Chapters all analyze the *informational content of interest rates* and study a common sequence which can be broken down into three components: i) We first investigate what kind of information agents learn from monetary policy; ii) We then analyze how do agents react to such information; iii) We finally infer how the Central Bank, anticipating agents' reaction to the information it shall divulge through monetary policy, decides to set interest rates or deliberates upon some institutional arrangement for monetary policy.

For concreteness, we proceed to relate in turn each of the three central chapters to the three steps informational economics sequence we have outline above. Chapter 3 interprets the informational content of interest rates as a process by which agents try to learn from the observed conduct of monetary policy the superior information on macroeconomic fundamentals the Central Bank might be endowed with. Hence this chapter captures the informational problem of step i) in the sequence by modeling the link between monetary policy and consumers' confidence. For instance, as a pure illustration of our findings in this regard, an abrupt reduction in interest rates might signal that the Central Bank forecasts a recession; hence, as a possible illustration for point ii) in the sequence in this context, agents curtail their consumption and investment behavior after observing a quick reduction in rates; hence, to describe point iii) in the sequence, the Central Bank might decide to bring into effect a gradual loosening of monetary policy, rather than an abrupt one which might destabilize markets and plummet consumers' confidence.

In the context of Chapter 4 on interest rate smoothing the informational content of interest rates of point i) in the above sequence consists of a process by which agents learn from the past conduct of monetary policy how informative a current change in the short-term rate is for the future path of interest rates. In fact, in this chapter agents employ a learning model for the yield curve and gradually learn over time by how much a revision in short-term rates should induce them to revise the medium and long portion of the yield curve. As of step ii) in the sequence, agents' beliefs on how informative interest rate changes are drive the slope and the steepness of the yield curve and the relationship between interest rates of various maturities; finally, in relation to point iii), in this chapter the Central Bank realizes that a path of partial adjustment for the level of short-run nominal rates and a low reversals to total changes ratio enable it to effect a large shift in the medium and portion of the yield curve with only a small shift in the short-one, which we show to be a desirable feature for the Central Bank under some stated, but perhaps not totally uncontroversial, assumptions.

The sequence also applies to the context of Chapter 2 on voting transparency in a Monetary Union. In fact, in the problem studies in this chapter agents must determine whether the representative of their country in the board of the Monetary Union's Central Bank has voted according to the partisan interests of the country that has appointed her, or instead, she has fulfilled her mission of acting as a sworn *super-partes* civil servant. This is the information agents learn in reference to point i) on the informational content of interest rates. The issue of voting secrecy in a Monetary Union was central in the heated debate between Buiter (Buiter 1999) and Issing (Issing 1999). We assume that under voting secrecy the behavior of members in the Central Banking panel is *observable* but not *verifiable*, so that, in relation to point ii) on how agents react to such information, we note the feature of our model by which Central Bankers are forced by agency problems to serve partisan interests under Transparent Voting (when individual voting records are revealed) but not under Secret Voting, exactly as claimed by the ECB. We show that there is a higher amount of macroeconomic volatility under Transparent Voting than under Secret Voting because transparency implies the supremacy of the median voter, unlike voting secrecy. Relating to point iii) on how the informational problem analyzed affects monetary policy, we show in Chapter 2 that Transparent Voting induces in our model a greater symmetry in supply shocks across member countries as it forces firms in a given industry to locate widely across the Monetary Union to try to hedge against the volatility in macroeconomic fundamentals that such voting transparency regime entails.

This, in turn, impacts the decision of the Central Bank as to what voting transparency regime to choose, since voting secrecy is welfare superior holding the asymmetry of output supply shocks constant, but voting transparency implies a lower degree of asymmetry of output supply shocks than voting secrecy.

1.1.2 Further Unifying Elements

A second methodological dimension unifies the three central chapters of the thesis. All the three central chapters employ some simple game-theoretic interaction framework in a macroeconomic setting. It must be admitted that the technical framework employed is always a simple one. Chapter 2 analyzes some simple Nash equilibria concepts in the context of two different regimes for voting transparency. Chapter 3 employs a highly stylized and simple signaling game theoretic framework which the Central Bank solves through the Cho-Kreps refinement criterion (Cho and Kreps 1987) to determine the optimal trade-off between inducing agents to behave in a pro-cyclical fashion by affecting their expectations (which, in itself, only acts to propagate and amplify the initial shock and to lessen the effectiveness of monetary policy) and allowing the cost of borrowing to move sharply in a counter-cyclical manner. Finally, Chapter 4 presents a learning yield curve model, so that the Central Bank must bear in mind that whenever setting policy it teaches agents at each stage how to react in future to the implemented choice for the short-run rate.

A third unifying theme for the three central chapters lies in the fact that they all aim to draw qualitative conclusions on some specific institutional aspect. We do not present calibrated and fully specified models and do not aim to write general frameworks which can deliver a simple optimal rule. Instead, we focus on studying in each chapter the implications of a specific effect. Therefore, rather than studying a universal optimal monetary policy rule, we rather view policy-makers as having a wealth of possible contradictory models in their mind while having to choose what specific effect is most important at any given point in time.

The remaining portion of this introductory chapter plays a double duty. On the one hand, we aim to define the three research questions addressed by the thesis. On the other hand, we offer some intuitive insights as to what aspects of the questions our investigation emphasizes. A thorough discussion of how our findings relate to the existing literature on each research question is deferred to the introductory section of each individual chapter.

1.2 Should Individual Voting Records be Published in a Monetary Union?

The unique arrangement of voting secrecy adopted by the European Central Bank has sparked a heated debate between Willem Buiter (Buiter 1999), at the time a member of the MPC and strongly critical of such arrangement, and Ottmar Issing, the chief economist of the ECB, who deems such provision to be welfare rising (Issing 1999). A wider debate on the issue of voting secrecy in a Monetary Union has ensued which motivates the analysis of Chapter 2.

The rationale for voting secrecy advanced by the ECB (Issing 1999) states that voting members of the ECB Governing Council would find *partisan pressure* irresistible and would, absent an arrangement of voting opacity, be unable to fulfill their role of sworn *super-partes* civil servants. It is often noted such a claim is not immune to criticism even at its descriptive level. In fact, actions by members at the ECB Governing Council would seem at least to be *observable* even under voting secrecy, although they might not be *verifiable* since individual voting records cannot be proven and hence can be discussed only at an informal level. This is so for ECB Governing Council's Meeting are attended by over thirty professional observers. Is the lack of verifiability of an individual voting record sufficient to insulate members of the ECB from partisan pressures? This question seems legitimate, though we choose not to tackle it.

Instead, we accept the ECB's statement that voting transparency induces partisan behavior at its face value and study its analytical consequences in a simple economic geography framework. In fact, while models of monetary policy in a nation-state economy usually abstract from the geographic structure of the macroeconomic framework, students of monetary policy in a Monetary Union cannot abstain from setting the economy in space because of the implicit admission by the ECB that in a Monetary Union policy-makers' incentives risk being affected by partisan considerations.

We assume that there exist three regions in our setting, each equally represented in the panel of the Central Bank. Each region specializes in a given industry, but also hosts, in a smaller proportion, the two other industries in which the other two countries of the Union specialize.

Hence, in this setting, output supply shocks are asymmetric, the more so the more each industry locates predominantly in the country in which it enjoys a comparative advantage, as assumed by Krugman (Krugman 1991). The Central region has the special feature of being the one each other country most resembles in terms of industrial structure.

We set the analysis at two different levels. We first wonder whether voting secrecy is optimal when the industrial structure is exogenous to how monetary policy is conducted. This first level and theoretically unsophisticated level of the analysis seems to most resemble the operative framework considered by the European Commission (Commission of the European Communities 1999). This first level of the analysis provides also some useful benchmark results but the second level of the analysis of this chapter, to which we now turn attention, is more subtle.

The second level of the analysis starts by considering this question: Is the issue of industrial structure exogenous to monetary policy? Krugman was the first to address this issue (Krugman 1991) and to answer such question in the negative. We also argue that the chosen regime for monetary policy has the theoretical effect of affecting the problem of industry location inside a Monetary Union. While Krugman argued that the microeconomic fact of external economies of scale induces a more specialized pattern of location in a Monetary Union, we compare and contrast the resulting pattern of location under voting secrecy and voting transparency. We argue in the context of a general equilibrium model that it is, in theory, possible that the choice of voting transparency regime, by affecting, as we show, the volatility of output, might also affect firm's incentives to locate widely as opposed to specializing production in a single region. Hence the industrial structure of the Monetary Union is endogenous to the choice of voting transparency regime, which, we show, has important welfare consequences.

These considerations clarify the setting for our analysis. The two frameworks developed in Chapter 2 allow us to investigate a number of detailed research questions. Is voting transparency welfare superior for the Monetary Union as a whole if we let the pattern of industrial location be, at the first level of the analysis, exogenous to the choice of voting transparency regime? We start our analysis by illustrating the perhaps trivial initial result that, under exogenous industrial location, secret voting is more welfare superior the more industrial structure differs across countries of the Monetary Union. However, even when industrial location is held to be exogenous to the choice of monetary policy regime, is voting secrecy welfare rising for all individual regions of the Monetary Union?

The answer to this question is, instead, ambiguous even at the first level of the analysis. In fact, secret voting is optimal even for the Center when its supply shocks bear the same covariance to the East as to the West; however, we show under some stated conditions that secret voting, though being welfare superior for the Union as a whole, might not be incentive compatible for a majority of the members of the Monetary Union. But is the assumption that the pattern of industrial location is exogenous to the adopted voting transparency regime justified?

We study this question by constructing a simple general equilibrium framework in which the choice of location by each firm is endogenous to the choice of voting rule for monetary policy. We find that the choice of transparent voting over secret voting has the effect of reducing the asymmetry of supply shocks across the various regions of the Monetary Union. We offer some intuition for this result. Transparent voting implies that the median voter always gets her first best choice, so that the chosen interest rate does not reflect the preferences of the country which happens to be out-voted in each contingency. This implies that volatility in inflation and output is higher under voting secrecy, as we show in the general equilibrium model.

Let us draw an analogy with financial economics to understand this result. Why would the investor be induced not to hold solely the stock that delivers the highest expected return? Or, in the context of our analysis, why would a given industry be induced not to solely locate in the area where it enjoys a comparative advantage? It is a common finding in financial economics that the investor might want to diversify her portfolio to reduce the variance of her consumption across various stochastic states of the world. Similarly, each firm might want to hedge macroeconomic risk by spreading its location widely. In this vein, we show that transparent voting increases macroeconomic volatilty in a single region by forcing members of the ECB's Governing Council to neglect the stabilization needs of countries that are out of cycle with the macroeconomic conditions experienced by the median voter country.

Is therefore voting secrecy always welfare rising once the industrial structure of the economy is made endogeous in our model ? The model we develop points to the fact that the answer to this question might be, rather surprisingly, an ambiguous one. In fact, we show that voting transparency, while welfare inferior holding constant the asymmetry of output supply shocks, might induce a greater degree of symmetry for supply shocks across member countries than what would be observed under voting secrecy. Conclusively, given that voting transparency induces greater symmetry in output supply shocks, the welfare comparison between the two voting rules may be ambiguous, even if we accept the ECB's claims at face value. We now turn attention to a second research question.

1.3 Gradualism, Interest Rate Smoothing and the Reversals to Total Changes Ratio

Central Banks are often accused of adjusting monetary policy *too little and too late* in response to forecasted macroeconomic shocks. This remark is prompted by the dual observation, whose account in the literature we summarize in the introduction to Chapter 4, that: (i) Central Banks smooth interest rate changes so that interest rates follow a partial adjustment mechanism; ii) and that, in the words of Goodhart ((Goodhart 1997), p.1): *"instead of adjusting interest rates by a large enough jump whenever inflation begins to deviate from its desired path, the authorities prefer to make relatively small changes... the consequence is therefore a series of relatively small interest rates changes in the same direction"*.

We interpret throughout the thesis these two observations to define the term *interest rate smoothing behavior*, which captures the concept that interest rates seem to some students of monetary policy excessively smooth in the face of the volatility in macroeconomic data and forecasts.

Our chosen approach emphasizes that interest rate smoothing behavior can arise even if the Central Bank does not have an explicit objective to smooth interest rate changes. In fact, a variety of models in the interest rate smoothing literature we discuss do not assume that the Central Bank has an explicit interest smoothing objective, but rather aim to show that interest rate smoothing behavior arises as the result of some considerations other than a concern to minimize the change in the level of interest rates.

In fact, students of interest rate smoothing aim to investigate whether such behavior has some optimal properties so as to dismiss the claim that, as argued by a number of authors (*inter alia* Goodhart (Goodhart 1997), Ball (Ball 1999) and Rudebusch (Rudebusch 1998)), interest rate smoothing behavior can be held in some regimes to be responsible for such a considerable welfare loss that one might wonder whether Central Banks view interest rate smoothing as a desirable objective *per se*, rather than being an optimal procedure through which output and inflation stabilization is best accomplished.

At a broader level, we believe that two research questions should be central to the literature of interest rate smoothing: i) Does interest rate smoothing lessen the Central Bank's ability to carry out inflation or output stabilization policy effectively?; ii) Can a framework be produced in which interest rate smoothing is optimal even if the Central Bank does not have an explicit objective to smooth interest rates?

It might be useful to motivate these two research questions by developing some empirical observations before providing an insight about the framework developed to address these two questions in Chapter 4.

We believe that two important empirical observations motivate the literature on interest rate smoothing. At a first level, it is often observed that the lagged level of the interest rate seem to be an important determinant of the current interest rate (see, *inter alia*, Clarida et al. (Clarida, Gali, and Gertler 1999), Woodford (Woodford 1999) and Sack et al. (Sack and Wieland 2000)). Such statement is often tested by specifying the following augmented Taylor-rule model for the nominal interest rate level i_t:

$$i_t = \rho \, i_{t-1} + (1-\rho)\Big[(rr^* + \pi_t) + \alpha(\pi_t - \pi^*) + \beta y_{t-1}\Big]; \tag{1.3.1}$$

The notation is defined as follows: rr^* captures the long-run equilibrium level of the real-rate (held to be exogenous); the other terms represent the deviation of inflation π_t from its target π_t^* and the logarithm level for the output gap y_{t-1}. The traditional Taylor rule is nested by this specification and can be obtained by setting $\rho = 0$.

Clarida et al. (Clarida, Gali, and Gertler 1999) indicate in their survey of the literature that estimates for ρ for the US economy vary across a spectrum ranging from 0.8 to 0.9. Confirming this result, Sacks et al. ((Sack and Wieland 2000),p.208) report in their survey of the interest rate smoothing literature that the finding of partial adjustment in the setting of the short-term interest rate is: *" greater than what can be attributed to the systematic policy responses to persistence in output and inflation fluctuations.. and is robust to other specifications, such as rules that respond to forecasts"*. The observation that the nominal interest rate follows a partial adjustment process is held by the literature to indicate that Central Banks, rather than adjusting interest rates via a one-off jump reflecting all the available macroeconomic information, follow a policy of adjusting interest rates gradually to a given expected target level- which is continuously re-assessed.

There is a second important source of evidence that points to interest rate smoothing behavior by Central Banks. This is developed by Goodhart (Goodhart 1997) by constructing a ratio between *reversals* (defined as all interest rate changes of opposite sign to the last interest rate change implemented) to total changes in the nominal interest rate instrument for monetary policy. We summarize and slightly update his findings in Table 1.1, which documents that nominal interest rate changes tend to be implemented through a series of adjustments in the same direction in line with the observation by Goodhart we have previously reported.

A high example of the tendency for reversals to be much less frequent than continuations is the conduct of monetary policy undertaken by the FED in 2001 when ten *continuations* movements have taken place.

Two important qualifications are in order, which we develop in greater detail in the introduction to Chapter 4. One could believe that *interest rate smoothing behavior* is only due to the autoregressive structure of macroeconomic shocks, which might display a serially correlated pattern. However, as a first response to this criticism note that a specification in the vein of (1.3.1), or a forecast based specification, would control for the relevant measure of inflation or output gap, so that the fact that shocks to output and inflation are highly positively serially correlated should be reflected in the coefficients on of these terms in (1.3.1) (α and β respectively) rather than on the coefficient ρ of the partial adjustment term.

Secondly, we could also develop a theoretical argument to understand why the serial correlation of shocks to macroeconomic fundamentals, such as inflation, does not in itself bias the ratio of continuations to reversals in interest rate changes in favor of

Source: Goodhart (Goodhart 1997) updated by author's computations

	Ratio of Reversals to Total Changes
UK Base Rate (05/1997-15/10/2001)	1: 7.33
EURO Refinance Rate (01/2000-09/2001)	1: 9.00
US FF Target Rate (1974-9, 1984-92)	1: 9.05
US Discount Rate (1974-9,1984-92)	1: 7.40
UK Base Rate (1974-9,1984-92)	1: 3.88
German Discount Rate (1974-9, 1984-92)	1: 6.90
Japanese Discount Rate (1974-9,1984-92)	1: 6.25
Japanese Overnight call rate (1974-9,1984-92)	1: 2.61
Australian Rediscount Rate (1974-9,1984-92)	1: 10.84
US Discount Rate (1962-95)	1: 5.00
UK Base Rate (1974-95)	1: 4.26
German Discount Rate (1974-95)	1: 7.33
Japanese Discount Rate (1974-95)	1: 9.00
Japanese Overnight Call Rate (1960-95)	1: 2.88
Australian Rediscount Rate (1974-93)	1: 6.93
Australian call money rate (1984-94)	1: 3.17

Table 1.1: Ratio of Reversals to Total Changes

continuations. Note that monetary policy affects inflation with a lag of such length that Central Bankers often target a two-years ahead inflation forecast. Therefore, the Central Bank must anticipate at any time the persistence and correlation in, for illustration, inflationary shocks when setting interest rates since it knows that it needs to conduct monetary policy in a forward looking manner. Hence, interest rates should *respond only to news* and monetary policy should not be affected by stale information. But news are, by definition, white-noise and hence should not impart a serially correlated pattern to interest rate changes.

The literature on *interest rate smoothing behavior* is reluctant to assume that Central Bankers make systematic mistakes in the conduct of monetary policy by acting *too little and too late*. Also, such literature tries to develop some accounts for interest rate smoothing without assuming that such behavior is an explicit objective of the Central Bank. A more detailed survey of the interest rate smoothing literature is developed in the introductory section of Chapter 4. However, a brief synopsis of three main family of models of the interest rates smoothing literature might help putting the analysis of Chapter 4 into context.

A first account for interest rate smoothing emphasizes model uncertainty, as illustrated, among others, by Brainard (Brainard 1967) and Wieland (Wieland 1998). However we explain in the introduction to Chapter 4 that such family of models, while being very interesting, relies on the assumption that all stochastic shocks to inflation are strictly multiplicative in the instrument of monetary policy, so that a large interest rate movement induces more uncertainty than a smaller one. Furthermore, this family of models imposes a strong restriction on the sign of the third derivative of the Central Bank's loss function and is not robust to the lag structure of the transmission mechanism we generally observe.

A second class of models which might be relevant to this problem (see, for instance, Orphanides et al. (Orphanides and Wieland 1998) and Smets (Smets 1991)) studies the implications of data uncertainty, which is pervasive in monetary policy. This class of models is successful in explaining why monetary policy does not react immediately to a large change in the measure of a macroeconomic fundamental. This is so for the Central Banker knows that macroeconomic measurements are more volatile than macroeconomic data because data is noisy. It is often noted, however, (see for instance (Sack and Wieland 2000), p.218) that it has not been proved to date that this kind of models can even theoretically account for partial adjustment in interest rates given the certainty equivalence properties of the setting usually employed in the analysis, as we explain in detail in Section 4.1. Hence, models of data uncertainty can explain why interest rates are smoother than fundamentals. However, this family of models can neither explain why the current interest rate is a function of the lagged one nor why continuations are more frequent than reversals.

The analysis of interest rate smoothing that we derive in Chapter 4 belongs to third family of models emphasizing the forward looking nature of agents expectations and the importance of the shape of the yield curve. To this family belong the models of Woodford (Woodford 1999) and Levin et al. (Levin, Wieland, and J.Williams 1999). The model developed in Chapter 4 bears some similarities to Woodford's analysis, but our results were independently derived. Moreover, while Woodford's results study the problem under commitment (a regime in which the Central Bank commits to a given policy rule), our analysis is developed under discretion (a regime in which the Central Bank sets its policy at each time only for the current period).

The starting point for our analysis, as for the analysis of all models in this third family, rests on the observation that the importance of short-term interest rates is only of second order. In fact, investments decisions tend to be based on the medium and long portion of the yield curve, as observed by Goodfriend (Goodfriend 1991). Hence, short-term rates are important to the extent they can affect the medium and the long portion of the yield curve.

More specifically, our analysis in Chapter 4 aims to answer two questions. Does interest rate smoothing behavior imply that Central Banks act too little and too late? And can the observation that the relevant measure of monetary policy lies in the medium and long portion of the yield curve, rather than in the short-term rate, account for a partial adjustment mechanism in the setting of interest rates?

We start the analysis by constructing a learning model of the yield curve whereby agents employ the historical path of short-run rates and the historical correlation of interest rate changes to determine the slope and the steepness of the yield curve. We first propose a model agents might employ to determine forward rates.

We assume that the term-structure theory of interest rates holds, so that agents determine the yield curve by viewing any long-term bond as a composite index of all the forward rates that mature before the given bond. The term structure theory of interest rates, then, allows us to employ our forward rate model to derive a yield curve model via arbitrage conditions, a quite common pricing strategy in financial economics (Bjork 1998).

We interpret the credibility of monetary policy as being represented by the Central Bank's capability of affecting a large movement in the medium and long portion of the yield curve with a relatively small change in the current short-run interest rate. For this to happen in our studied adaptive learning model the Central Bank must have conducted monetary policy in the past by carrying out a low reversals to total continuations ratio. In fact, if the Central Bank carries out interest rate changes via a number of serially correlated movements, agents learn that a current increase in the interest rate, for instance, signals that a further wave of tightening moves is to come. Hence agents would then attach a very high signaling value to interest rate changes.

We therefore find that a positive pattern of historical serial correlation in interest rate changes implies that the Central Bank can bring into effect a large movement in the long portion of the yield curve with a small change in short-run rates, suggestive of the fact that a low reversal to total changes ratio and interest rate smoothing behavior do not necessarily imply an excessively timid response to macroeconomic shocks.

We then build on the results of the first part of the analysis to study the qualitative behavior of monetary policy. We assume that the Central Bank incorporates in its loss function the level of the short-run rate, to which we give a number of justifications, ranging from a concern for the indebtness of the private sector, a concern for mortgage holders and the aim not to induce agents to have to unnecessarily economize on cash. The marginal disutility from a high interest rate is therefore assumed to be increasing in the level of the interest rate itself. We show that this assumption implies that it is welfare rising for the Central Bank to be able to affect a large movement in long-term rates with a small movement in short-term ones.

We show that in this context the short-term rate is increasing in its lag and in its lagged rate of change so that monetary policy exhibits a partial adjustment mechanism. We also find that the short-term rate display in our model a short-run path dependent behavior.

We show that our findings rely on the assumption that the Central Bank attaches some disutility to a high level of the short-term rate. We also stress that all models of interest rate smoothing do not seem to be particularly robust. Therefore, each family of models must be interpreted as contributing to a wide debate rather than providing a final solution to the question of why Central Banks seem to smooth interest rates.

Chapter 4 is not the only chapter of the thesis touching upon the debate on gradualism and the reversals to total changes ratio. We argue that this debate is somewhat connected to our third research question, to which we now turn attention.

1.4 Interest Rates as a Vehicle of Information: The Economic Consequences of the Degree of Information Transparency Adopted

Members of the Federal Open Market Committee of the FED reach their policy decisions after having been presented with a rich variety of macroeconomic forecasts, including the FED's macro model predictions at various horizons for output and inflation, the forecasts of staff members (which might differ from those of the macro model) as well as the macroeconomic forecasts formulated by all the other members of the FOMC.

This wealth of information is not reported in the minutes and is shared with the public with a lag of five years (Romer and Romer 2000). The procedure of relative information secrecy adopted by the FED has been challenged by the private sector in the early 80's though a lawsuit which was unsuccessful. Among the arguments advanced by the FED in its legal defense lied a concern that full information transparency could induce unnecessary volatility in financial markets (Goodfriend 1986).

Institutional arrangements as to extent upon which the Central Bank shares its information with agents vary widely across institutions. The Bank of England, for instance, provides the public with a diagramatic illustration of its models' forecasts in the monthly *Inflation Report*. Moreover, the minutes of the MPC's meetings often refer to various forecasted scenarios contingent upon the given interest rate path the Committee is discussing.

However, even when disclosure of information is complete Central Bankers seem to be aware that the public holds the Central Bank's actions to be a vehicle of information about the macroeconomic outlook (in fact, this is might be so for for the public is trying to infer from the chosen path for interest rates how the Central Bank interprets a large number of forecasts that are often contradictory or characterized by very wide confidence intervals). Such concern for the informational content of interest rates is, as a pure illustration of a point often recurring in the minutes of the MPC's meetings, expressed in the minutes of the November 1998 meeting when the merits of a fifty-five basis points cut were weighted against the arguments in favor of a larger seventy-five basis point reduction in interest rates ((Bank of England 1998), point 36): *"Notwithstanding the opportunity to* *explain any policy decision in the following week's Inflation Report, there could well be [if the large seventy-five basis points cut is implemented] a prolonged effect on perceptions of the Committee's assessment of the outlook, with a risk that people, business and markets mistakenly concluded that the Committee knew something that it had not disclosed about the outlook"*.

The study of how decisions by the Central Bank act as a vehicle of information as to the assessment of the macroeconomic outlook becomes particularly interesting when Central Banks are endowed with *asymmetric and superior information* as to the path of macroeconomic fundamentals. The testing of whether Central Banks are indeed endowed with asymmetric information is recent, but in an interesting recent study David and Chirstina Romer (Romer and Romer 2000) argue that the FED's macroeconomic forecasts are vastly superior to those produced by the private sector. They also show that such superior forecasting performance is due to a genuine forecasting advantage held by the Central Bank on the path of macroeconomic variables, rather than stemming from the FED's superior knowledge of its own future actions, as we discuss in greater detail in the introduction to Chapter 3.

In this context, a recent strand of literature has emerged studying the economic properties of information secrecy. Its central question might be summarized as follows: Is information transparency welfare rising? We clarify the terms of the debate by defining *information transparency* as the polar case in which the Central Bank shares promptly and fully with the public its current assessment of the outlook for macroeconomic fundamentals. Instead, *information secrecy* is held in the literature to represent the opposite polar case in which agents are unaware of the information observed by the Central Bank, though some learning can occur through the observations of the Central Bank's actions. Therefore, the literature on information transparency operates in the following framework: i) the Central Bank is assumed to be endowed with asymmetric information on macroeconomic fundamentals; ii) agents try to learn such information via monetary policy when information secrecy is adopted; instead, if information transparency holds, agents' assessment of the macroeconomic outlook is independent of the actions undertaken by the Central Bank.

A more detailed summary of the information transparency literature is deferred to the first section of Chapter 3. However, we can anticipate at the descriptive level a pattern to be discerned in this literature: information secrecy is welfare diminishing whenever models of time-consistency tend to be employed in the analysis (see, for instance, Faust and Svensson (Faust and Svensson 2000) and Geraats (Geerats 2000)); on the other hand, information secrecy is welfare superior when monetary policy is not held to have an inflationary bias and the sole objective of the Central Banker lies in output stabilization (to this class of models belongs the work of Cukierman (Cukierman 1999) and of Gersbach (Gersbach 1998)).

We follow the central intuition of the information secrecy literature which assumes that the Central Bank enjoys superior information on the magnitude of an output shock and study a signal extraction problem in Chapter 3. We develop a model in which consumers' confidence (that is, agents' forecasts of their disposable income) is re-assessed in the context of a signaling game by the private sector by observing the behavior of monetary policy. The novelty of our analysis lies in allowing agents to condition their income expectations (rather than solely their inflationary expectations as often assumed in the information transparency literature) upon the observed conduct of monetary policy. In other words, we let the *animal spirits of the investors* be rationally affected by monetary policy as agents let their assessment of their future disposable income be affected by the signals they receive from the Central Bank.

Consider the following scenario to gain an insight of the intuition behind the model we develop in Chapter 3. The Central Bank, endowed with superior information on the path of macroeconomic fundamentals (Romer and Romer 2000), forecasts a sharp recession. It might be tempted to lower rates very aggressively to stimulate investment. And yet, it instead acts cautiously, postponing an easing of monetary policy or implementing it via a partial adjustment gradual mechanism.

Such behavior is brought about as the Central Bank realizes that agents might deduce from an aggressive easing of monetary policy that a sharp recession is forecasted. Hence a large change in interest rates would lead consumption and investment decisions to be revised in a sharply pro-cyclical fashion.

This scenario summarizes the intuition behind the model. We develop a simple signaling game in which the Central Bank, acting as the sender, observes output shocks and sets monetary policy. Agents, acting as the receivers, condition their consumption decisions upon their belief on the information observed by the Central Bank, which they partially infer observing monetary policy.

The signaling game is solved by backwards induction. We employ the Cho-Kreps refinement criterion (Cho and Kreps 1987) to impose some structure on agents' beliefs. A number of interesting results emerge as we argue in Chapter 3 that a very wide number of questions can be studied in this framework.

First of all, can it be argued that the Central Bank adopts a gradualist policy approach in the setting of our model because it seeks to stabilize agents expectations? And if this is true, would it also be the case that interest rates are less responsive to macroeconomic shocks under information secrecy and asymmetric information than they would be under either information transparency of symmetric information? A comment in passing to this effect is carried out in the conclusion of an influential survey (Clarida, Gali, and Gertler 1999) which seems to harmonize well with our results. In fact, we answer this first question in the affirmative by showing that in our model under asymmetric information and information secrecy interest rates are less volatile that under full information.

Is information transparency welfare rising in our model? We show that such question is ambiguous in our setting and try to define conditions under which information transparency is welfare rising. We also employ our model to study what are some possible economic effects of publishing the minutes of the Interest Rate Setting Panel.

Note that the quote from the November 1998 MPC meeting reported at the beginning of this section is suggestive of limit pricing behavior in the fashion of the effects first explored by Milgrom and Roberts (Milgrom and Roberts 1982) in games of imperfect information. In fact, the quote seems to suggest that at least some members of the MPC (which we can define in that context as having observed a relatively moderate recessionary or deflationary shock) believe that they needed to set monetary policy in such a way as to avoid inducing agents to believe that the Central Bank has instead observed a very large recessionary or deflationary shock. In the language of game theory, the quoted excerpt from the 1998 November MPC meeting indicates that the Central Bank, when it is in reality a low deflation type, must separate itself from a high deflation type by adopting a limit pricing behavior.

Is such limit pricing effect supported by our model? We pursue this line of research and show that in some cases our model does indeed produce limit pricing behavior.

Finally, can the model account for the low reversals to total changes ratio commonly observed in the conduct of monetary policy? We are not able to fully analyze this question, but however we illustrate an example which suggests that the assumption that the Central Bank holds asymmetric information has the effect of producing some bias in favor of continuations. This concludes our discussion of the three main research themes of the thesis, whose structure we now discuss.

1.5 The Structure of the Thesis

The strategy followed in organizing the thesis is the following. We illustrate in each central chapter one of the three models analyzed. Chapter 2 studies the problem of voting secrecy in a Monetary Union. Chapter 3 analyzes information secrecy, while finally Chapter 4 investigates a model of partial adjustment for the setting of the nominal interest rate.

Each model focuses the analysis focuses narrowly on a specific effect rather than studying each research question in a general setting. Therefore, we can attempt to translate our conclusions into some suggestive policy insights only with great caution and with many important qualifications. We devote therefore the conclusive chapter of the thesis to relating the results of the central chapters to the debate on various policy questions and to trying to critically assess our findings.

Chapter 2

Should Individual Voting Records be Published in a Monetary Union? The Location of Industry and the Choice of Voting Transparency Regime

Abstract

We compare the welfare impact of Transparent Individual Voting as opposed to Secret Individual Voting for the setting of Monetary Policy in a Monetary Union. We accept at face value the ECB's claim that Transparent Voting forces members of the Interest Rate Setting Panel to be influenced by partisan interests rather than by Monetary Union wide considerations and set the analysis in a simple economic geography framework. We study the question at two levels.

If the issue of industry location is held exogenous to the Monetary Voting process, we find that: i) Secret Voting is the more welfare superior the more the industrial structure differs across countries of the Monetary Union; ii) Secret Voting is optimal even for the Center when its supply shocks bear the same covariance to the East as to the West; iii) under some stated conditions, Secret Voting, tough being welfare superior for the Monetary Union as a whole, is not incentive compatible for a majority of the member countries.

We then construct a simple general equilibrium framework in which the choice of location by industry is endogenous to the choice of the Voting Transparency Regime for monetary policy. We find that the choice of Transparent Voting over Secret Voting has the effect of reducing the asymmetry of supply shocks across the various regions of the Monetary Union, suggesting that the welfare comparison between the two voting rules may be ambiguous, even if we accept the ECB's claims at face value.

KEYWORDS: VOTING TRANSPARENCY IN A MONETARY UNION, THE LOCATION OF INDUSTRY IN A MONETARY UNION, HEDGING OF MACROECONOMIC RISK IN A MONETARY UNION.

2.1 Introduction

Students of Transparency in Central Banking usually identify three dimensions in assessing how observable and verifiable the procedures of Monetary Policy are for the public: *Information Transparency, Goal Transparency and Voting Transparency* (Winkler 1999).

Voting Transparency, the subject of this chapter, measures the extent upon which the public is informed about the voting behavior (and its motivation) of each member of the monetary policy setting body of the Central Bank. We observe a sharp contrast between the European Central Bank and Central Banks of other OECD countries in terms of the transparency of the voting procedures adopted.

In fact, on the one hand, the Bank of England, the FED and the Bank of Japan all publish, with a varying degree of delay, individual voting records after the Policy Committee meets. The voting record of individual members of the Bank of England's MPC is generally divulged to the public two weeks after the vote is cast; the Bank of Japan, under the New Bank of Japan Law legislated in 1998, publishes individual voting records eight weeks after the Policy Board has met and, while the bulk of the minutes is non-attributed, members dissenting from the majority vote are bound to explain the reasons of their dissent in an attributed section of the notes. Furthermore, the Federal Open Market Committee of the FED publishes individual voting records seven weeks after the meeting has taken place.

Such pattern of disclosure of individual voting records contrasts sharply with the arrangement chosen by the European Central Bank (Commission of the European Communities 1999). In fact, the European Central Bank plans to disclose individual voting records with a lag of seventeen years.

The attempt by the European Central Bank to keep individual voting records secret has given rise to a heated debate between Willem Buiter (Buiter 1999), at the time member of the MPC and strongly critical of such arrangement, and Ottmar Issing, the chief economist at the ECB who supports the provision for up-keeping secrecy on individual voting records (Issing 1999).

Is there any reason why Voting Secrecy may have some welfare rising consequence in a Monetary Union? Architects of the ECB (Issing 1999) answer this question in the affirmative by claiming that, without Voting Secrecy, executive members of the ECB

Governing Council would be under an irresistible pressure to only act according to *the partisan interests of the member country that has appointed them* rather than fulfill their mission as *sworn super-partes civil servants*.

The aim of this chapter is to analyze the welfare comparison between Voting Transparency and Voting Secrecy in a Monetary Union taking the statement by the ECB at face value and abstracting from other considerations that might affect the choice of what Voting Transparency Regime to adopt in a Monetary Union. However, before we take the ECB's statement at face value for the remainder of the paper, we would like to develop some caveats.

While some authors believe that the Central Bank is indeed able to uphold Voting Secrecy if it chooses to (Gersbach and Hahn 2000), others have observed that, given the sheer number of agents in attendance to Governing Council's meetings, individual voting records are, in fact, *observable* since leaking cannot be ruled out (Buiter 1999). Individual voting records might be *observable* (in the sense that national governments might know the individual voting records of members of the Governing Council), but they are unlikely to be *verifiable* (in the sense that the principal cannot prove its knowledge of the agent's behavior in a court).

Furthermore, the sheer observability of individual voting records may be weakened by the fact that agents may agree on the outcome of the Interest Rate Setting Panel's meeting in an informal manner before the meeting takes place. Moreover, the Committee could reach its decision without taking a formal vote, as stated by the ECB's chairman Duisenberg at a press conference (Duisenberg and C.Noyer 2000).

We, therefore, explore throughout the remainder of the chapter the consequence of assuming that the Voting Secrecy Regime (from henceforth the regime in which individual voting records are not published) is an analytically different regime to the Voting Transparency Regime (from henceforth the regime in which individual voting records are published). In this vein, the study of voting transparency is now giving rise to a small literature of which we now give a brief account.

Sibert (Sibert 1999) studies the welfare impact of publishing individual voting records in the context of an overlapping generations model for members of the Central Bank Policy Committee. Policy-makers' preferences over the relative dis-utility attached to output and inflation are assumed to be dictated by a type which the public cannot observe. Sibert finds that social welfare is lower when individual voting records are published since this this gives an incentive for a *dove* type to initially dress up as a *hawk* and then take the public by surprise at a later stage in the game, exacerbating the time-consistency problem of monetary policy.

Gersbach and Hahn (Gersbach and Hahn 2000), instead, analyze an effect by which welfare is higher under Voting Transparency. Components of the Monetary Panel are assumed in their framework to differ in their competence, defined as their efficiency in forecasting output supply shocks. Voting Transparency allows the public to gradually learn which members of the Committee are efficient in setting policy so that incompetent members can be replaced. Therefore, the competence of the members of the Policy Committee is higher under Voting Transparency.

The authors, however, do not consider the possibility that members of the Committee may themselves learn over time who the most efficient policy-makers are, and therefore emulation of the most efficient members of the Committee by the less efficient ones may act, under Voting Secrecy, as a surrogate to Voting Transparency in ensuring that the most efficient policy-makers set monetary policy.

The focus of this chapter consists of analyzing at two different levels the consequences of the ECB's statement that Voting Transparency induces partisan behavior in a Monetary Union. We, therefore abstract from the issues studied by the aforementioned authors, and instead specialize the analysis to the case of the choice of the optimal voting regime in a Monetary Union.

We therefore set out our analysis in a very simple *spatial framework*, in which the degree of asymmetry of output supply shocks depends upon the pattern of geographic specialization of each industry, as argued by Krugman (Krugman 1991). This assumption has some important consequences.

First of all, setting the analysis in spatial terms implies that not all countries enjoy the same ex-ante probability of being pivotal in the interest rate setting decision since a Center-Periphery structure might hold. Therefore, some regions (the Center) are more likely to act as median voters than others (the Periphery) for the output supply shocks that hit the Center are likely to be most correlated ones to the shocks hitting the other regions in the Monetary Union.

Secondly, as proposed by Krugman (Krugman 1991), the choice of industry on where to locate, which dictates the degree of asymmetry of output supply shocks across member countries of the Monetary Union, might be a variable endogenous to the choice of Monetary Policy Regime.

In fact, Krugman argues that the United States witness a greater degree of geographic specialization of industry than Europe: the existence of increasing returns to scale implies that the removal of trading barriers induces firms in the same industry to specialize production in the same single region, rather than spreading widely their productive activities into several regions.

We wonder, in the context of our analysis, whether modeling the choice of industrial location by firms as being affected by the conduct of monetary policy has important consequences for the choice of Transparency Voting Regime in a Monetary Union. Therefore, our analysis is carried out at two levels.

We first hold in Section 2.2 the decision of industrial location by firms *exogenous* to the choice of Transparency Regime and, hence, also *exogenous* to the conduct of monetary policy and the institutional arrangements which regulate the Central Bank. This is the first level of our analysis. At this level we find that Voting Secrecy is optimal in a Monetary Union, the more so the more specialized is industrial location and we construct a measure of the welfare cost of Voting Transparency.

We also find that Voting Secrecy may be under some stated conditions welfare optimal even for the Center, in spite of the fact that the Center is likely, as we show, to act as the median voter under Voting Transparency. We characterize this finding by analogy to the purchase of an insurance policy by which agents trade-off obtaining their first best outcome in most contingencies against diminishing the volatility of their welfare across different states of the world.

We also characterize conditions under which Secret Voting, while being welfare optimal for the Union as a whole, is preferred by a majority of member countries.

We then take the analysis to a deeper level in Section 2.3, where we let the choice of industrial location by firms be *endogenous* to how the Central Bank chooses to conduct monetary policy in a Monetary Union. We analyze the problem by constructing a simple general equilibrium model, which extends the framework of Blanchard and Kiyotaki (Blanchard and N.Kiyotaki 1987).

We show in the context of the simple general equilibrium model we develop that the choice of Voting Secrecy over Voting Transparency has the effect of increasing the degree of asymmetry of the output supply shocks hitting the member countries of the Monetary Union. We interpret the result by analogy with a portfolio choice problem.

In fact, we show that Voting Transparency in a Monetary Union makes output, aggregate demand, labor and employment more volatile in each region that under the Secret Voting Regime. For this reason, agents have a greater incentive under Voting Transparency to spread industrial location widely across all regions of the Monetary Union, rather than specializing production in the region where production is more efficient for a given industry.

We then devote the final section to conclusions and a final discussion.

2.2 The Choice of Monetary Policy Voting Transparency Rule in a Monetary Union when Industrial Structure is held Exogenous

2.2.1 The Framework

We develop in this section the first of the two models of this chapter analyzing the framework for the problem of the choice of Voting Transparency Regime in a Monetary Union in the context of *asymmetric supply shocks* when industrial location is held exogenous to monetary policy. We first state in Section 2.2.1.1 the functional form of the loss function and the Phillips curve to which monetary policy is subjected in each region of the Monetary Union.

We then proceed in Section 2.2.1.2 to establish how each country would have conducted monetary policy had it stayed independent of the Monetary Union. This is a useful benchmark to analyze in later stages of the chapter the voting pattern of each member country in the Monetary Union.

Finally, we describe in Section 2.2.1.3 the two alternative rules for Voting Transparency whose welfare comparisons we analyze throughout the chapter.

2.2.1.1 The Basic Assumptions:

Each member country of the Monetary Union is averse to instability in the level of the price index and to deviations of output from a bliss point $k\hat{y}$. The loss function L_i for country i takes the following form *a' la' Barro and Gordon* (Barro and Gordon 1983) (though we emphasize that our results, as we later show in Remark 2.2.4, do not rely upon the existence of a time-consistency problem in monetary policy and therefore we could let $k_i = 1$ in the following loss function without affecting the conclusions of this section):

$$L_i\left(y_i, \pi_i; \beta, k_i\right) = (y_i - k_i\hat{y})^2 + \beta(\pi_i)^2, k_i \geq 1; \tag{2.2.1}$$

The variables y_i and π_i denote denote the logarithm of output and inflation respectively. The parameter k_i is usually assumed to be greater or equal to one reflecting the fact that imperfect competition and distortionary taxes imply that the Walrasian equilibrium of output and employment is sub-optimal as the marginal revenue for the representative good is generally above marginal cost.

We impose the very important further assumption that k_i is the same across countries and set $k_i = k\ \forall i$. We would like to emphasize that the results of the model we develop here may not generalize to the case in which k_i varies across countries, as we later note in Remark 2.2.4.

While each country is free to set π_i independently before joining a Monetary Union, a common Monetary Policy implies that that an unique inflation rate (the instrument of monetary policy in the setup of the model) is chosen for all countries in the Monetary Union. While this is a common assumption used in the analysis of a Monetary Union (see, for instance, Dixit and Lambertini (Dixit and L.Lambertini 2000), Monticelli (Monticelli 2000), Krugman (Krugman 1995) and Pagano (Giavazzi and Pagano 1988)) the assumption may lack realism as: i) the instrument of monetary policy cannot be realistically deemed to be inflation itself and the assumption that monetary policy controls inflation directly is made for analytical simplicity when analytical interest lies in studying problems connected to short-run output-inflation trade-offs; ii) even though purchasing power parity would predict that in a Monetary Union inflation should be constant across countries, deviations from purchasing-power parity are possible, at least in the short-run, as long as the cost of arbitraging goods across countries is higher than the inflationary differential.

However, the conclusions of this section are robust to a relaxation of the assumption that inflation is constant across countries, as we argue in Appendix A.2.

Monetary Policy feeds upon to output in each country through the following Phillips curve:

$$y_i(\pi_i, \hat{y}, \pi^e; \gamma) = \hat{y} + \gamma(\pi_i - \pi_i^e) + z_i; \tag{2.2.2}$$

We indicate the expected level of inflation in each country by π_i^e. As wages are assumed to be sticky, if inflation is higher (lower) than predicted, then the ex-post real wage falls (rises) taking output above (below) its unconditional expectation level $\hat{y}$.

Finally, z_i is a stochastic white-noise stochastic term that captures the impact of supply shocks in each country, which we define more precisely in (2.2.3).

We assume that there are only three member countries in the Monetary Union, denoted as East, Center and West. The three output supply shocks hitting each country correlate and take the following form:

$$z_i(d_{i,j}, \epsilon_e, \epsilon_c, \epsilon_w) = \begin{cases} z_e & = d_{e,e}\epsilon_e + d_{e,c}\epsilon_c \\ z_c & = d_{c,e}\epsilon_e + d_{c,c}\epsilon_c + d_{c,w}\epsilon_w \\ z_w & = d_{w,c}\epsilon_c + d_{w,w}\epsilon_w \end{cases} , 0 \leq d_{i,j} \leq 1; \tag{2.2.3}$$

We now turn attention to defining and interpreting all the terms in (2.2.3). We assume that the economy consists only of three industrial sectors, denoted as the Eastern Industry, the Central Industry and the Western Industry respectively. Each industry is subject to a stochastic output shock ϵ_i, drawn from an independent distribution such that:

$$\epsilon_i = \begin{cases} \bar{\epsilon} \ with \ Prob \ \frac{1}{2} \\ -\bar{\epsilon} \ with \ Prob \ \frac{1}{2} \end{cases} i = e, c, w; \tag{2.2.4}$$

The parameter $d_{i,j}$ is a weight reflecting the fraction of industry j located in region i. For example, assume that eighty per cent of industry e is located in the East and twenty per cent in the Center and a positive supply shock of magnitude $\overline{\epsilon}$ occurs to industry e. Then, neglecting all other factors, a supply shock of magnitude 0.8 $\overline{\epsilon}$ will occur to the East, while a positive supply shock of 0.2 $\overline{\epsilon}$ hits the Center.

We now impose some restrictions on the parameters $d_{i,j}$. First of all, we posit that each region is endowed with a comparative advantage in one of the three industries, and each industry predominantly locates in the industry where it enjoys its comparative advantage, so that:

$$d_{i,j=i} \geq \frac{1}{2} \ \forall i; \tag{2.2.5}$$

This restriction implies that, for instance, the industry labeled as Eastern Industry locates predominantly in the East, where it enjoys its comparative advantage (so that $d_{e,e} \geq \frac{1}{2}$). The same applies to the Central Industry (locating predominantly in the Center as $d_{c,c} \geq \frac{1}{2}$) and to the Western Industry.

Secondly, we wish to impose a restriction on the set of parameters $d_{i,j}$ to ensure that each of the two peripheral countries is likely to experience supply shocks more closely synchronized to those occurring to the Center rather than to those occurring to the other peripheral country. To achieve this, we assume that:

$$\begin{aligned} COV(z_e, z_c) &> COV(z_e, z_w); \\ COV(z_w, z_c) &> COV(z_w, z_e); \end{aligned} \tag{2.2.6}$$

The restriction imposed in equation (2.2.6), employing (2.2.3) to compute the various expressions for the covariance function and bearing in mind that z_i^2 is assumed to be constant across industries, turns out to imply:

$$\begin{aligned} d_{e,e}d_{c,e} + d_{e,c}d_{c,c} &> d_{e,c}d_{w,c} \\ d_{w,w}d_{c,w} + d_{w,c}d_{c,c} &> d_{w,e}d_{e,w}; \end{aligned} \tag{2.2.7}$$

We finally assume that the variance of supply shocks is constant across countries, which (bearing in mind again that the we assumed that the idiosyncratic shocks to each industry z_e, z_c, z_w have the same variance) requires the following condition to hold:

$$d_{i,e} + d_{i,c} + d_{i,w} = 1 \ \forall i; \tag{2.2.8}$$

Note, finally, that equation (2.2.8) and (2.2.3) jointly imply that $(z_i)^2 = VAR(z_i) = (\overline{\epsilon})^2 \ \forall i$.

2.2.1.2 The Conduct of Monetary Policy Under Independence from the Monetary Union:

We recall in this section the standard analysis for the conduct of monetary policy applying to each country if it stays independent of the Monetary Union. While the analysis of this brief sub-section is not an original research result, this benchmark will turn to be useful when we determine in Section 2.2.1.3 the impact of the choice of Voting Transparency Regime on agents' voting behavior.

Each country would, under independence, set its inflationary rate as to minimize the loss function of (2.2.1) subject to (2.2.2) which, taking agents' inflationary expectations as given, leads to the following reaction curve for the Central Bank linking the choice of inflation to the inflation rate expected by agents:

$$\pi_i(\pi^e; z_i, \hat{y}, k) = (\beta + \gamma)^{-1} \Big[\hat{y}(k-1) + \gamma \pi^e - z_i \Big]; \tag{2.2.9}$$

Agents form rational expectations and therefore aim to avoid systematic mistakes in predicting expected inflation. The only procedure to avoid systematic mistakes is to form a prediction of inflation π^e such that $E(\pi_i \big| \pi^e) = \pi^e$ along the reaction function of (2.2.9), implying that:

$$E(\pi_i) = \frac{1}{\beta} \Big[\hat{y}(k-1) \Big]; \tag{2.2.10}$$

Substituting the rational expectations inflation rate of (2.2.10) into equation (2.2.9) the optimal choice of inflation turns out to be:

$$\pi_i^* = \frac{\hat{y}(k-1)}{\beta} - (\beta+\gamma)^{-1} z_i; \tag{2.2.11}$$

Ploughing back the optimal inflation rate of (2.2.11) into the Phillips curve of (2.2.2) we now determine output:

$$y_i^* = \hat{y} + \frac{\beta}{\beta+\gamma} z_i; \tag{2.2.12}$$

Finally, to determine the value of the loss function we substitute (2.2.12) and (2.2.11) into (2.2.1) and, after rearrangement, we obtain:

$$\begin{aligned} L_i^*(y_i, \pi_i, \beta, \gamma) = {} & \left(\frac{1+\beta}{\beta}\right) [\hat{y}(k-1)]^2 + \\ & + \frac{\beta(\beta+1)}{(\beta+\gamma)^2} (\overline{\epsilon})^2 + \left[\frac{\beta-1}{\beta+\gamma}\right] 2\hat{y}(1-k) z_i; \end{aligned} \tag{2.2.13}$$

Having fixed ideas on how monetary policy is conducted under independence of the Monetary Union, we now proceed to defining how voting procedures affect monetary policy in a Monetary Union.

2.2.1.3 Two Regimes for Voting Transparency Rules in a Monetary Union:

Does the choice of Voting Transparency Regime affect the individual voting behavior of Members of the Interest Setting body in a Monetary Union?

We analyze in this section the impact of voting transparency on the determination of Monetary Policy in a Monetary Union. Two different rules are considered.

If individual voting records are published, then we define the voting regime as being characterized by *Voting Transparency.* Otherwise, when the vote on interest rates of individual members of the Monetary Policy Setting Panel is kept secret (as in the case of the European Central Bank), we define the Voting Regime as being one of *Secret Individual Voting.*

To understand the likely effect of Secret Individual Voting, we recall the rationale given by the ECB for opting to keep individual voting behavior secret. It is claimed by the European Central Bank that individual voters, were their individual voting records to be published, would be affected by *partisan interests only.* In fact, the ECB maintains that, were individual voting records divulged to the public, the representative of each

Member Country would only take macroeconomic conditions in her country of origin into account when deciding on how to cast her vote in the Interest Rate Setting Council.

Instead, the ECB claims, Individual Voting Secrecy insulates members of the Interest Rate Voting Body from pressures stemming from member countries. As a result, Individual Voting Secrecy is maintained to allow Members of the Voting Council to fulfill their mission as sworn *super-partes* civil servants. In other words, Individual Voting Secrecy allows policy-makers to take into account the Pan-European Macroeconomic scenario and to behave as *benevolent social planners.*

Is the case in favor of Secret Individual Voting depicted by the European Central Bank plausible? We do not really tackle this issue. Instead, we are interested in exploring some implications of the ECB's view on Voting Transparency in a Monetary Union, which we accept at face value in the following assumption:

Assumption 2.2.1. ***(Impact of Voting Transparency):*** *Accepting the ECB's statements at face value, we assume that the publication of Individual Voting Records (the Voting Transparency Regime) forces individual members of the Interest Rate Voting Council to be affected only by the interests of the Member Country they represent.*

Instead, under Individual Voting Secrecy individual members of the Interest Rate Voting Body behave in a super-partes manner and weight by the same factor the welfare of all member countries of the Monetary Union.

We assume that Interest Rates are set by a panel composed by three members, so that all the three regions of the Monetary Union are equally represented.

We now proceed to characterize the impact of the choice of voting secrecy rule adopted on the conduct of monetary policy.

Monetary Policy Under Voting Secrecy:

Assumption 2.2.1 implies that under Voting Secrecy all members of the Interest setting body aim to maximize welfare at the Pan-Union level, as we explicate in the following remark:

Remark 2.2.1. ***(Monetary Policy under Voting Secrecy):*** *If individual voting is secret, then each member of the Interest Rate Voting Council sets inflation as to minimize the Union-wide loss function, so that inflation is chosen according to:*

$$\pi^{sv,*} = \ argmin \ \frac{1}{3}\Big[L_e\left(y_e,\pi;\beta,z_e\right) + L_c\left(y_c,\pi;\beta,z_c\right) + L_w\left(y_w,\pi;\beta,z_w\right)\Big]; \qquad (2.2.14)$$

It is interesting to also note that the voting rule applied under Secret Voting implies that all members of the Interest Rate Setting Panel are predicted to always agree on the same choice of interest rates.

In fact, note that we have assumed that Voting Secrecy implies that all interest rate voters, regardless of the country they represent, act to minimize the same loss function of (2.2.14). Furthermore, for the purposes of our model all policy-makers are assumed to believe in the same simple model of the economy, as outlined in Section 2.2.1.1 and Section 2.2.1.2.

Therefore, if we accept the ECB's statement that Voting Secrecy leads policy-makers to be guided by Union-wide considerations only, all interest rates setting decisions should be expected to be taken unanimously unless policy-makers disagree on what is the appropriate model of the economy, which might seem plausible even though it is a consideration from which we abstract in this chapter.

We not turn attention to deriving the monetary policy rule which would hold under Voting Secrecy. To this end, the following remark shall be very useful.

Remark 2.2.2. ***(Maximization Equivalence Problem under Secret Voting):*** *In the regime of Secret Voting the choice of inflation after that a set of supply shocks* (z_e, z_c, z_w) *is observed is equivalent to the choice of inflation under the One Country Independent monetary policy problem outlined in Section 2.2.1.2 setting the realized supply shock to take magnitude* $\frac{z_e+z_c+z_w}{3}$. *This implies that the solution to:*

$$\pi^{sv,*} = \ argmin \ \frac{1}{3}\Big[L_e\left(y_e,\pi;\beta,z_e\right) + L_c\left(y_c,\pi;\beta,z_c\right) + L_w\left(y_w,\pi;\beta,z_w\right)\Big]; \qquad (2.2.15)$$

is equivalent to:

$$\begin{aligned}\pi^* = &\ argmin \ L_i\left(y_i,\pi_i;\beta,\overline{z} = \left(\frac{z_e+z_c+z_w}{3}\right)\right)\\ =&argmin\left(\hat{y} - k\hat{y} + \gamma(\pi-\pi^e) + \frac{z_e+z_c+z_w}{3}\right)^2 + \beta\left(\pi\right)^2;\end{aligned} \qquad (2.2.16)$$

Where inflation in each country π_i is now restricted to taking a common value across all members of the Currency Union. Moreover, the loss function for individual countries takes the form stated in equation (2.2.1).

Proof. Let us first write out fully the function to be minimized according to equation (2.2.15):

$$\begin{aligned}
&\frac{1}{3}\left[L_e\big(y_e,\pi;\beta,z_e\big)+L_c\big(y_c,\pi;\beta,z_c\big)+L_w\big(y_w,\pi;\beta,z_w\big)\right]=\\
&+\frac{1}{3}\big(\hat{y}(1-k)+\gamma(\pi-\pi^e)+z_e\big)^2+\frac{1}{3}\big(\hat{y}(1-k)+\gamma(\pi-\pi^e)+z_w\big)^2+\\
&\frac{1}{3}\big(\hat{y}(1-k)+\gamma(\pi-\pi^e)+z_w\big)^2+\beta(\pi)^2;
\end{aligned} \tag{2.2.17}$$

We expand the quadratic expressions and exploit the assumption that $z_e^2 = z_w^2 = z_c^2$,and after re-arranging, the above expression simplifies to:

$$\begin{aligned}
&\frac{1}{3}\Big[L_e\left(y_e,\pi;\beta,z_e\right)+L_c\left(y_c,\pi;\beta,z_c\right)+L_w\left(y_w,\pi;\beta,z_w\right)\Big]=\\
&+\left(\hat{y}(1-k)+\gamma(\pi-\pi^e)\right)^2+\sigma_\epsilon^2+\frac{2}{3}\left[\hat{y}(1-k)+\gamma(\pi-\pi^e)\right]\\
&\left[z_e+z_c+z_w\right]+\beta\left(\pi\right)^2\\
=&\left(\hat{y}(1-k)+\gamma(\pi-\pi^e)+\frac{z_e+z_c+z_w}{3}\right)^2+\beta\left(\pi^2\right);
\end{aligned} \tag{2.2.18}$$

□

This remark implies that the determination of $\pi^{sv,*}$, the optimal inflation rate under Voting Secrecy, follows, once the output supply shock is appropriately re-weighted, a procedure analogous to the optimal setting of monetary policy for a country independent of the Monetary Union. In fact, the welfare maximization problem of (2.2.14) is solved by letting $z_i = \frac{z_e+z_c+z_w}{3}$ in equation (2.2.11), so that the rate of inflation chosen by the Central Bank of the Monetary Union under Voting Secrecy is:

$$\pi^{sv,*} = \frac{\hat{y}(k-1)}{\beta} - (\beta+\gamma)^{-1}\frac{z_e+z_c+z_w}{3}; \tag{2.2.19}$$

Monetary Policy Under Voting Transparency:

We study in this section the conduct of monetary policy under Voting Transparency. First of all, we notice that Assumption 2.2.1 implies that under Voting Transparency any member of the Monetary Panel only aims to maximize welfare in her country of origin. Therefore, the representative of country i aims to set π^{tv}, the inflation rate under Voting Transparency, as to minimize:

$$L_i\left(y_i, \pi^{tv}; \beta, k_i\right) = (y_i - k_i\hat{y})^2 + \beta(\pi^{tv})^2, k_i \geq 1; \tag{2.2.20}$$

Note that the rate of inflation π^{tv} is not set by any country independently, but it is rather set equal to the preference of the median voter in the Interest Rate Setting Panel of the Monetary Union's Central Bank.

Denote with π_i^{tv} the rate of inflation for which the representative of country i votes under Voting Transparency. Output in each country depends on the un-anticipated component of π, the rate of inflation for the Monetary Union, according to the following Phillips curve:

$$y_i(\pi, \hat{y}, \pi^e; \gamma) = \hat{y} + \gamma(\pi^{tv} - \pi^e) + z_i; \tag{2.2.21}$$

To derive π_i^{tv}, notice that each voter sets π_i^{tv} as a function of π^e by minimizing (2.2.20) subject to (2.2.21), so that the following set of reaction function for the vote cast by each voter obtains:

$$\pi_i^{tv}(\pi^e; z_i, \hat{y}, k) = \begin{cases} \pi_e^{tv} = & (\beta+\gamma)^{-1}\left[\hat{y}(k-1) + \gamma\pi^e - z_e\right]; \\ \pi_c^{tv} = & (\beta+\gamma)^{-1}\left[\hat{y}(k-1) + \gamma\pi^e - z_c\right]; \\ \pi_w^{tv} = & (\beta+\gamma)^{-1}\left[\hat{y}(k-1) + \gamma\pi^e - z_w\right]; \end{cases} \tag{2.2.22}$$

Each voter has a different reaction function since the magnitude of the output shock z_i varies across regions. We now proceed to establish according to which reaction function is monetary policy set. Since Monetary Policy is set by majority voting, then the vote cast by the median voter in (2.2.22) determines the reaction function followed by the Central Bank of the Monetary Union.

Let us denote by z^{mv} the median value of the output supply shock occurring among the three country-specific shocks z_i, z_e, z_w. Equation (2.2.22) shows that the median voter (the representative of the country voting for the median value of π_i^{tv}) is the representative of the country hit by z^{mv} as long as k, β *and* $\hat{y}$ are, as assumed, the same for all member countries. The reaction curve of the median voter takes therefore the following form:

$$\pi_{mv}^{tv}(\pi^e) = (\beta + \gamma)^{-1}\Big[\hat{y}(k-1) + \gamma\pi^e - z^{mv}\Big]; \tag{2.2.23}$$

Agents determine π^e by using rational expectations. Though the identity of the median voter is not known ex-ante, agents know that $E\left[z_i\right] = 0\ \forall i$ and therefore also expect $E\left[z^{mv}\right] = 0$. The only rational expectation estimator for π^e is one such that, just as under independence, $E[\pi_{mv}^{tv}|\pi^e] = \pi^e$. This implies that the only rational expectations rate of inflation is equal to:

$$\pi^e = E\left[\pi_{mv}^{tv}(\pi^e)\right] = \frac{1}{\beta}\left[\hat{y}(k-1)\right]; \tag{2.2.24}$$

By ploughing back (2.2.24) into (2.2.23) we can finally determine the conduct of monetary policy under Voting Transparency in the next remark:

Remark 2.2.3. ***(Monetary Policy Under Transparent Voting):*** *Let us denote by z^{mv} the median value among the output supply shocks z_e, z_w, z_c hitting each member country. Under Transparent voting the median voter, who has experienced the output supply shock z^{mv}, sets the rate of inflation to:*

$$\pi^{tv,*} = \frac{\hat{y}(k-1)}{\beta} - (\beta+\gamma)^{-1} z^{mv}; \tag{2.2.25}$$

This concludes the set up of the model used throughout Section 2.2. We are now ready to study the welfare implications of the choice of voting regime when industrial structure is held exogenous to monetary policy.

2.2.2 Welfare Comparisons among Different Voting Transparency Regimes when the East and the West are Equally Asymmetric to the Center

We study in this section some welfare implications of the choice of Voting Transparency Regime after imposing a further restriction on the structure of supply shocks of equation (2.2.3). In fact, we assume throughout this section that:

$$COV\left(z_e, z_c\right) = COV\left(z_w, z_c\right); \tag{2.2.26}$$

This implies that the pattern of industrial location is such that the degree of asymmetry in supply shocks between the Center and the West is the same as between the Center and the East. This assumption is to be relaxed in Section 2.2.3.

2.2.2.1 Assumptions about the Structure of the Supply Shocks:

We now parametrize the structure of the output supply shocks in equation (2.2.3) in the following way:

$$z_i(d_{i,j}, \epsilon_e, \epsilon_c, \epsilon_w) = \begin{cases} z_e & = \left(\frac{3}{4} + M\right)\epsilon_e + \left(\frac{1}{4} - M\right)\epsilon_c \\ z_c & = \left(\frac{1}{4} - M\right)\epsilon_e + (\frac{1}{2} + 2M)\epsilon_c + \left(\frac{1}{4} - M\right)\epsilon_w; \\ z_w & = \left(\frac{1}{4} - M\right)\epsilon_c + \left(\frac{3}{4} + M\right)\epsilon_w, M \leq \frac{1}{4}; \end{cases} \tag{2.2.27}$$

It should be recalled that the coefficient on ϵ_j for country i represents the share of industry j that locates in country i. For instance, the parametrization of (2.2.27) implies that a share of $\frac{1}{4} - M$ of the Western Industry locates in the Center while a share of $\frac{3}{4} + M$ of Western Industry locates in the West itself.

Note also that all the restrictions on supply shocks of equations (2.2.5),(2.2.6), and (2.2.8) are satisfied. In fact, the parametrization of $d_{i,j}$ of equation (2.2.27) implies that industry j locates predominantly in the region $i = j$ where it enjoys its comparative advantage; furthermore, each peripheral region experiences supply shocks that correlate by a greater degree with the Center than with the other peripheral region; finally, the variance of supply shocks is the same for each country.

What is the role of M in the parametrization of supply shocks of (2.2.27)? To throw light on this question we introduce the following definition:

Definition 2.2.1. ***(Index of Geographic Symmetry of Output Supply Shocks):*** *We define the index of geographic symmetry of industrial structure as:*

$$I_{gs} = COV(z_e, z_c) + COV(z_e, z_w) + COV(z_c, z_w); \tag{2.2.28}$$

Such index, a measure of the symmetry in industrial structure and in the macroeconomic output supply shocks across the three regions, is decreasing in M.

To verify that the index I_{gs} is indeed decreasing in M we employ (2.2.27) to calculate the following set of covariances:

$$\begin{cases} COV(\epsilon_e, \epsilon_c) = & \sigma_\epsilon^2 \left(\frac{5}{16} - \frac{1}{2}M - 3M^2\right); \\ COV\left(\epsilon_w, \epsilon_c\right) = & \sigma_\epsilon^2 \left(\frac{5}{16} - \frac{1}{2}M - 3M^2\right); \\ COV(\epsilon_w, \epsilon_e) = & \sigma_\epsilon^2 \left(\frac{1}{4} - M\right); \end{cases} \tag{2.2.29}$$

It therefore follows from (2.2.29) that:

$$I_{gs} = COV(z_e, z_c) + COV(z_e, z_w) + COV(z_c, z_w) = \sigma_\epsilon^2 \left(+1 - 2M - 6M^2\right); \tag{2.2.30}$$

Thus, the lower is M, the more similar is the industrial structure across countries, and hence the more symmetric is the set of output supply shocks hitting the Monetary Union's Member Countries.

2.2.2.2 Optimal Choice of Voting Transparency Regime:

It is interesting to ask at this stage what is the optimal choice of Voting Transparency Regime given the structure of output supply shocks posited in (2.2.27) and assuming that industrial location is exogenous to the choice of Transparency Voting Regime. Is Voting Transparency costly in the sense that it is welfare diminishing? And what determines the magnitude of its welfare cost? We first define a welfare measure of the cost of Voting Transparency and then show that such cost is non-negative and rising in the asymmetry of output supply shocks.

Definition 2.2.2. ***(The Cost of Voting Transparency):*** *The cost of voting transparency is defined as:*

$$E[C_{tv}] = E\Big[L_{e,c,w}(\overline{z}; \pi = \pi^{tv,*}) - L_{e,c,w}(\overline{z}; \pi = \pi^{sv,*})\Big]; \tag{2.2.31}$$

where we also define:

$$L_{e,c,w}(\overline{z}; \pi) = \frac{1}{3}\Big[L_e(z_e, \pi) + L_c(z_c, \pi) + L_w(z_w, \pi)\Big]; \tag{2.2.32}$$

We now seek to study the magnitude and the sign of the cost of Voting Transparency.

Proposition 2.2.1. ***(Cost of Voting Transparency and the Location of Industry):*** *Assuming the location of industry is exogenous, the cost of Voting Transparency* *is non-negative and increasing in M, or equivalently the cost of Voting Transparency increasing in the degree of asymmetry in industrial structure across the three regions of the Monetary Union.*

Proof. The first part of the proposition is trivially proved by noticing again that by definition:

$$\pi^{sv,*} = \ argmin \ \frac{1}{3}\left[L_e\left(y_e,\pi;\beta,z_e\right)+L_c\left(y_c,\pi;\beta,z_c\right)+L_w\left(y_w,\pi;\beta,z_w\right)\right]; \tag{2.2.33}$$

This so since we have assumed that under Voting Secrecy the Central Bank behaves as if it were a benevolent social planner wishing to set monetary policy with the view of minimizing the welfare loss function across the whole Monetary Union.

We now aim to show that $E\left[C_{vt}\right]$ is increasing in M to proof the second part of the proposition. To verify this we start by carrying out a Taylor expansion of $L_{e,c,w}(\overline{z};\pi)$ around $\pi^{sv,*}$:

$$L_{e,c,w}(\overline{z};\pi) \approx L_{e,c,w}(\overline{z};\pi^{sv,*})+\left(\pi-\pi^{sv,*}\right)\frac{\partial L_{e,c,w}(\pi=\pi^{sv,*})}{\partial\pi}+\frac{\left(\pi-\pi^{sv,*}\right)^2}{2}\frac{\partial^2 L_{e,c,w}(\pi=\pi^{sv,*})}{(\partial\pi)^2}; \tag{2.2.34}$$

Note that:

$$\frac{\partial L_{e,c,w}(\pi=\pi^{sv,*})}{\partial\pi}=0;$$

This is so since $\pi^{sv,*}$ is by definition the value of inflation that minimizes the Monetary Union wide loss function $L_{e,c,w}(\pi,\overline{z})$. If we evaluate (2.2.34) letting $\pi=\pi^{tv,*}$ we then obtain:

$$C_{tv}=L_{e,c,w}(\overline{z};\pi=\pi^{tv,*})-L_{e,c,w}(\overline{z};\pi=\pi^{sv,*})=\frac{\left(\pi^{tv,*}-\pi^{sv,*}\right)^2}{2}\frac{\partial^2 L_{e,c,w}(\pi=\pi^{sv,*})}{(\partial\pi)^2}; \tag{2.2.35}$$

Note, furthermore, that by ploughing (2.2.2) and (2.2.10) into (2.2.1) we verify that:

$$\frac{\partial^2 L_{e,c,w}(\pi=\pi^{sv,*})}{(\partial\pi)^2}=2(\beta+\gamma); \tag{2.2.36}$$

Note also that equations (2.2.19) and (2.2.25) imply that the difference in the inflation rate across the Transparent and the Secret Voting Regime is:

$$\left(\pi^{tv,*}-\pi^{sv,*}\right)=-(\beta+\gamma)^{-1}\left[\frac{z^e+z^c+z^w}{3}-z^{mv}\right]; \tag{2.2.37}$$

To prove the statement of the proposition substitute (2.2.36) and (2.2.37) into (2.2.35) and take expectations of the resulting expression to get:

$$E\Big[C_{vt}\Big] = \big(\beta+\gamma\big)^{-1}E\left[\Big(\frac{z^e+z^c+z^w}{3}-z^{mv}\Big)^2\right]; \tag{2.2.38}$$

We recall that z^{mv} denotes the output supply shock experienced by the country acting as the median voter under Voting Transparency. We now aim to show that $E\big(\pi^{tv,*}-\pi^{sv,*}\big)$ is actually rising in M. In fact, if this is true, then the expression for $E[C_{vt}]$ is also increasing in M.

We can use Table 2.1 to show that, indeed, this is the case. The second column of the table records the magnitude of the triplet of binomial output supply shocks $(\epsilon_e, \epsilon_c, \epsilon_w)$ occurring, respectively, to the Eastern, the Central and the Western industry. The third column illustrates the Union-wide average level $\overline{z}$ of such output shocks. Columns four to six illustrate the magnitude of the supply shocks z_e, z_c *and* z_w occurring in each region of the Monetary Union, which we have derived using the assumptions about the industrial structure in each country stated in (2.2.27). Finally, the last column indicates which country acts as the median voter in each contingency.

Using Table 2.1 for computation we can see that:

$$E\left[(\pi^{tv,*}-\pi^{mv,*})\right]^2 = E\left[\Big(\frac{z^e+z^c+z^w}{3}-z^{mv}\Big)^2\right] = \frac{3}{4}\big(\beta+\gamma\big)\big(\frac{1}{6}\overline{\epsilon}+2M\hat{\epsilon}\big)^2; \tag{2.2.39}$$

Therefore ploughing (2.2.39) into (2.2.38) we verify that the right hand side of (2.2.38) is rising in M implying that $E[C_{vt}]$ is also positive and rising in M, which concludes the proof. □

We have established that Secret Voting is welfare superior to Transparent Voting for the Union as a whole. However, is it also welfare superior for each individual region?

In fact, the Center happens to act *ex-ante* as the most likely median voter if Voting Transparency is implemented. This means that, in most cases, the Center gets its first best choice for monetary policy under Voting Transparency, whereas the same is not necessarily true under Voting Secrecy. In fact, under Voting Secrecy Monetary Policy is set with a view on stabilizing the macroeconomic cycle of the Monetary Union as a whole. Therefore, under Voting Secrecy, unlike under Voting Transparency, the Center

Case	$(\epsilon_e, \epsilon_c, \epsilon_w)$	$\overline{z}$	z_e	z_c	z_w	Median Voter
1.	$(\overline{\epsilon}, \overline{\epsilon}, \overline{\epsilon})$	$\overline{\epsilon}$	$\overline{\epsilon}$	$\overline{\epsilon}$	$\overline{\epsilon}$	All
2.	$(\overline{\epsilon}, \overline{\epsilon}, \underline{\epsilon})$	$\frac{\overline{\epsilon}}{3}$	$\overline{\epsilon}$	$0.5\overline{\epsilon} + 2M\overline{\epsilon}$	$-0.5\overline{\epsilon} - 2M\overline{\epsilon}$	C.
3.	$(\overline{\epsilon}, \underline{\epsilon}, \overline{\epsilon})$	$\frac{\overline{\epsilon}}{3}$	$0.5\overline{\epsilon} + 2M\overline{\epsilon}$	$-4M\overline{\epsilon}$	$+0.5\overline{\epsilon} + 2M\overline{\epsilon}$	E.,W.
4.	$(\overline{\epsilon}, \underline{\epsilon}, \underline{\epsilon})$	$-\frac{\overline{\epsilon}}{3}$	$0.5\overline{\epsilon} + 2M\overline{\epsilon}$	$-0.5\overline{\epsilon} - 2M\overline{\epsilon}$	$-\overline{\epsilon}$	C.
5.	$(\underline{\epsilon}, \overline{\epsilon}, \overline{\epsilon})$	$\frac{\overline{\epsilon}}{3}$	$-0.5\overline{\epsilon} - 2M\overline{\epsilon}$	$0.5\overline{\epsilon} + 2M\overline{\epsilon}$	$\overline{\epsilon}$	C.
6.	$(\underline{\epsilon}, \overline{\epsilon}, \underline{\epsilon})$	$-\frac{\overline{\epsilon}}{3}$	$-0.5\overline{\epsilon} - 2M\overline{\epsilon}$	$4M\overline{\epsilon}$	$-0.5\overline{\epsilon} - 2M\overline{\epsilon}$	E.,W.
7.	$(\underline{\epsilon}, \underline{\epsilon}, \overline{\epsilon})$	$\frac{\overline{-\epsilon}}{3}$	$-\overline{\epsilon}$	$-0.5\overline{\epsilon} - 2M\overline{\epsilon}$	$0.5\overline{\epsilon} + 2M\overline{\epsilon}$	C.
8.	$(\underline{\epsilon}, \underline{\epsilon}, \underline{\epsilon})$	$-\overline{\epsilon}$	-$\overline{\epsilon}$	- $\overline{\epsilon}$	- $\overline{\epsilon}$	All

Table 2.1: The Impact of Geographic Dispersion on Monetary Policy

cannot exploit its position as the most likely median voter to get its first best outcome. However, it turns out that Voting Secrecy is optimal also for the Center, as we show in the next Proposition.

Proposition 2.2.2. ***(Transparency Optimal for the Center):*** *The Center, in spite of being the most likely median voter under Voting Transparency, is better off with Secret Voting rather than with Transparent Voting.*

The Welfare gain for the Center from the choice of Secret Voting over Transparent Voting is diminishing in I_{gs}, the index of industrial structure symmetry across regions of the Monetary Union.

Proof. We aim to show that the expected loss function for the Center under Secret Voting is lower than under Transparent Voting, therefore we aim to prove that:

$$E\left[L_c(\pi^{tv,*}, z_c) - L_c(\pi^{sv,*}, z_c)\right] > 0; \tag{2.2.40}$$

Recall that $\pi^{*,tv}$ and $\pi^{sv,*}$ denote *the optimal choice for inflation that obtains under Secret and Transparent Voting* respectively.

We now carry out a second order Taylor expansion of the loss function $L_c(z_c, \pi)$ for the Center. We carry out the Taylor expansion around π^{*c}, with which we denote the optimal choice for inflation that would have occurred if the Center did not belong to a Monetary Union, but rather was free to set monetary policy independently:

$$L_c(\pi^{tv,*}, z_c) \approx L_c(\pi^{*c}) + (\pi^{tv,*} - \pi^{*c})\frac{\partial L_c(\pi^{*c})}{\partial \pi} + \frac{(\pi^{tv,*} - \pi^{*c})^2}{2}\frac{\partial^2 L_c(\pi^{*c})}{(\partial \pi)^2}; \qquad (2.2.41)$$

By an analogous procedure, we also approximate the loss function for the Center under Secret Voting:

$$L_c(\pi^{sv,*}, z_c) \approx L_c(\pi^{*c}) + (\pi^{sv,*} - \pi^{*c})\frac{\partial L_c(\pi^{*c})}{\partial \pi} + \frac{(\pi^{sv,*} - \pi^{*c})^2}{2}\frac{\partial^2 L_c(\pi^{*c})}{(\partial \pi)^2}; \qquad (2.2.42)$$

To simplify both equations (2.2.41) and (2.2.42), notice that since π^{c*} minimizes $L_c(z_c, \pi)$, it then obtains that:

$$\frac{\partial L_c(\pi^{*c})}{\partial \pi} = 0; \qquad (2.2.43)$$

Exploiting this knowledge, then the second term on the right hand side of both (2.2.42) and (2.2.41) cancels out. We can then subtract (2.2.42) from (2.2.41) and take expectations to write down the expression we wish to study in this proof:

$$E\left[L_c(\pi^{tv,*}, z_c) - L_c(\pi^{sv,*}, z_c)\right] = E\left\{\frac{\partial^2 L_c(\pi^{*c})}{\partial \pi}\left[\left[\pi^{tv,*} - \pi^{*c}\right]^2 - \left[\pi^{sv,*} - \pi^{*c}\right]^2\right]\right\}; \qquad (2.2.44)$$

The rest of the proof aims to calculate and sign the right hand side of this last expression. First of all, by substituting (2.2.2) and (2.2.10) into (2.2.1) and differentiating we can show that:

$$\frac{\partial^2 L_c(\pi^{*c})}{(\partial \pi)^2} = 2(\beta + \gamma); \qquad (2.2.45)$$

We not turn attention to the calculation of the other expressions in the right hand side of (2.2.44). The equilibrium value for inflation under independent monetary policy computed in (2.2.11) and the equilibrium value of inflation under Transparent Voting in the Monetary Union derived in (2.2.25) imply that:

$$E\left(\pi^{*c} - \pi^{tv,*}\right)^2 = E\left[\frac{z^{mv} - z^{*c}}{\beta + \gamma}\right]^2; \qquad (2.2.46)$$

It must be recalled that z^{mv} denotes the shock to output experienced by the median voter under the Transparency Voting Regime.

By inspection of Table 2.1 and by working through all the eight possible cases, we compute:

$$E\left(\pi^{*c} - \pi^{tv,*}\right)^2 = E\left[\frac{0.5\overline{\epsilon} + 6M\overline{\epsilon}}{4(\beta+\gamma)}\right]^2; \tag{2.2.47}$$

Similarly, exploiting (2.2.11) and (2.2.19) we obtain the result that the difference in the rate of inflation between the scenario in which the Center conducts monetary policy independently and one in which Secret Voting prevails in a Monetary Union is equal to:

$$E(\pi^{*c} - \pi^{sv,*})^2 = E\left[\frac{z^{sv,*} - z^{*c}}{\beta+\gamma}\right]^2; \tag{2.2.48}$$

By using Table 2.1 we can compute the expression above as:

$$E(\pi^{*c} - \pi^{sv,*})^2 = E\left[\frac{(\frac{1}{6}\overline{\epsilon} + 2M\overline{\epsilon})^2}{2(\beta+\gamma)} + \frac{(\frac{1}{3}\overline{\epsilon} + 4M\overline{\epsilon})^2}{4(\beta+\gamma)}\right]; \tag{2.2.49}$$

We now substitute (2.2.49) and (2.2.46)into equation (2.2.44) to obtain the expression we set out to derive:

$$E\Big[L_c(\pi^{tv,*}) - L_c(\pi^{sv})\Big] = (\overline{\epsilon})^2[2(\beta+\gamma)]^{(-1)}\Big[\frac{1}{48} + 3M^2 + 0.5M\Big]; \tag{2.2.50}$$

Such expression is positive for all values of M. The welfare loss for the Center from the choice of Transparency Voting over Secret Voting is positive and increasing in the magnitude of M. This concludes the proof.

□

The result might seem, upon first inspection, counter-intuitive. In fact, the Center is the most likely median voter under Voting Transparency, a regime under which it can get its first best choice for monetary policy in six cases out of eight, as Table 2.1 shows. Why would the Center opt for the Secret Voting Regime and in so doing surrender its status as the most likely median voter?

The intuition for the result rests on the fact that the Center prefers to buy insurance against being out-voted. In fact, the loss function is concave, which makes the Center risk averse. Note also that Transparent Voting involves a greater volatility in the value of the *ex-post* loss function that Secret Voting does.

This is so because Table 2.1 shows that, on one hand, in six cases out of eight the Center, acting as the median voter, implements its first best choice of monetary policy if the Voting Regime is one of Transparency.

However, consider sub-cases 6. and 8. in the table, in which the set of output supply shocks in each industry $(\epsilon_e, \epsilon_c, \epsilon_w)$ takes values $(\overline{\epsilon}, \underline{\epsilon}, \overline{\epsilon})$ and $(\underline{\epsilon}, \overline{\epsilon}, \underline{\epsilon})$ respectively. The Center would get out-voted in both these contingencies under Transparent Voting. Furthermore, monetary policy would in these cases turn out to be expansionary (contractionary) just when the main industry locating in the Center is hit by a positive (negative) output supply shock.

Instead, opting for Secret Voting acts as an insurance policy also for the Center. In fact, on the one hand under Secret Voting the Center is less likely to dictate the conduct of monetary policy. However, when the Center is hit, say, by a positive supply shock while the other regions in the Union are hit by a negative shock, Secret Voting implies that policy-makers have to act as benevolent social planners and attempt to stabilize macroeconomic fundamentals also in the Center (so that in this case monetary policy is less restrictive than it would have been under Voting Transparency for the Central Bank also weights the overheating risks faced by the Center). Instead, under Transparent Voting no attempt is made to incorporate the preferences of the out-voted countries into the policy dictated by the median voter.

How general is the result we have just discussed? It is clear that were the Center to act as the median voter in all circumstances, then the Center would always be better off under the Transparency Voting Regime. However, the Center is bound under Transparent Voting not to be able to act as the median voter in some contingencies in a framework in which there are three regions and the shocks to output in each industry take on a binomial value. For this reason Secret Voting acts as an insurance policy and might be welfare superior even for the Center.

We conclude this section by highlighting a tangential implication of our model, which contrasts with some results in the literature. This question may not be very central to our results, but it is worth emphasizing that neither the choice of Voting Transparency Regime nor the choice between joining a Monetary Union or conducting policy independently affect the inflationary bias of Monetary Policy in our model.

Remark 2.2.4. ***(No Inflation Bias Introduced by Voting Rules):*** *Unlike in previous research (see, for instance, Monticelli (Monticelli 2000)) we find that neither the choice of Voting rules nor the decision of entering in a Monetary Union affect the inflation bias of monetary policy. In other words, the expected rate of inflation is the same under Independent Monetary Policy, Monetary Policy in a Monetary Union with Transparent Voting and Monetary Policy in a Monetary Union under the Secret Individual Voting Regime. This also confirms that our results do not depend upon the existence of a time consistency problem in monetary policy and hold even if k=1.*

Proof. We aim to show that regardless of the choice of Monetary Policy Regime:

$$E\left[\pi\right] = \frac{\hat{y}(k-1)}{\beta}; \tag{2.2.51}$$

Notice that since $E[z_i] = 0 \ \forall i$:

$$E[\overline{z}] = E\left[\frac{z_e + z_c + z_w}{3}\right] = 0; \tag{2.2.52}$$

And by the same mechanism it is also true that:

$$E\left[z^{mv}\right] = 0; \tag{2.2.53}$$

Hence taking expectations of (2.2.19) and of (2.2.25) we verify that:

$$E\left[\pi^{sv,*}\right] = E\left[\pi^{tv,*}\right] = \frac{\hat{y}(k-1)}{\beta}; \tag{2.2.54}$$

By taking expectations of (2.2.11) we can see that also under Independent Monetary Policy: $E\left[\pi_i^*\right] = \frac{\hat{y}(k-1)}{\beta}$.

□

2.2.3 The Choice of Voting Transparency Regime in a Monetary Union with Two Centers and One Periphery:

We have previously established that, when $COV(z_e, z_w) = COV(z_w, z_c)$, Voting Transparency is welfare superior to Voting Secrecy *both* for the Union as a whole and for each individual region, including the Center.

How general is such conclusion? The question is particularly interesting if we imagine that members of the Monetary Union engage in some pre-play negotiation to design the rules of the Central Bank. Would the welfare superior Secret Voting Rule be actually implemented?

If $COV(z_e, z_c) = COV(z_w, z_c)$, Proposition 2.2.2 shows, all members of the Monetary Union, including the Center, will opt for Secret Voting Rules.

However, does the majority of Member Countries opt for Transparent Voting also when $COV(z_e, z_c) > COV(z_w, z_c)$? This is the question we address in this section.

2.2.3.1 Assumptions about the Structure of Supply Shocks:

We have so far assumed that both peripheral countries experience output supply shocks that have the same covariance to the output supply shocks experienced by the Center. We now relax this assumption and posit, instead, that:

$$COV(z_e, z_w) \geq COV(z_w, z_c); \tag{2.2.55}$$

Assumption 2.2.55 implies that the industrial structure of the East is more similar to the one of the Center than the one of the West is.

We now paramaterize the share of industry j in region i represented by $d_{i,j}$, so that the set of idiosyncratic output supply shocks of (2.2.3) takes the form:

$$z_i(d_{i,j}, \epsilon_e, \epsilon_c, \epsilon_w) = \begin{cases} z_e & = \left(\frac{3}{4} - D\right)\epsilon_e + \left(\frac{1}{4} + D\right)\epsilon_c, \ D \leq \frac{1}{4} \\ z_c & = \left(\frac{1}{4} + D\right)\epsilon_e + \frac{1}{2}\epsilon_c + \left(\frac{1}{4} - D\right)\epsilon_w, \ D \leq \frac{1}{4}; \\ z_w & = \left(\frac{3}{4} + D\right)\epsilon_w + \left(\frac{1}{4} - D\right)\epsilon_c, \ D \leq \frac{1}{4}; \end{cases} \tag{2.2.56}$$

Note that (2.2.56) satisfies restrictions (2.2.5), (2.2.6),(2.2.8). In fact, eq. (2.2.56) implies that industry j locates predominantly in the region i=j where it enjoys a comparative advantage; all regions enjoy the same variance of output supply shocks; the share of industry j in each region adds up to one aggregating over the three regions in the Monetary Union.

We employ (2.2.56) to calculate the covariance of output supply shocks across regions:

$$\begin{cases} COV(z_c, z_e) & = \sigma_\epsilon^2 \left[\frac{5}{16} + D - D^2\right]; D \leq \frac{1}{4}; \\ COV(z_c, z_w) & = \sigma_\epsilon^2 \left[\frac{5}{16} - D + D^2\right]; D \leq \frac{1}{4}; \\ COV(z_e, z_w) & = \sigma_\epsilon^2 \left[\frac{1}{16} - D^2\right]; D \leq \frac{1}{4}; \end{cases} \tag{2.2.57}$$

Note that (given that $D \leq \frac{1}{4}$) the covariance of output supply shocks between the Center and the East is rising in D, while the covariance of output supply shocks between the Center and the West is diminishing in D, which clarifies what is the role of the parameter D:

Remark 2.2.5. ***(Role of** D**):*** *Increasing D acts to make the East and the Center experience more symmetric supply shocks, while the Center and the West became more asymmetric so that as D increases we approach a model characterized by two very similar regions (two Centers) and one Peripheral Country hit by asymmetric supply shocks.*

We are now ready to study how the magnitude of D can affect the choice of voting rules.

2.2.3.2 On the Incentive Compatibility of Secret Secret Voting in a a Two-Centers One Periphery Monetary Union:

We aim to show in this section that, under some stated conditions, a majority of countries in the Monetary Union is better off with Transparent Voting if D is sufficiently high, as we specify in the next proposition.

Proposition 2.2.3. ***(Transparent Voting preferred by a Majority of Countries with Asymmetric Peripheral Countries):*** *When one of the two Peripheral Countries experiences supply shocks more symmetric to the shocks occurring to the Center than the other does, then a majority of countries in the Monetary Union favors Transparent Voting. This occurs if D is greater than a threshold value D_1^{th} so that the industrial structure of the Eastern and of the Central Region are sufficiently similar.*

Proof. The proof is in two parts. We first calculate under which conditions Transparent Voting is welfare superior for the East. We then go through the same exercise as to find what values for D make Transparent Voting welfare superior for the Center as well.

Case	$(\epsilon_e, \epsilon_c, \epsilon_w)$	$\overline{z}$	z_e	z_c	z_w	Median
1.	$(\overline{\epsilon}, \overline{\epsilon}, \overline{\epsilon})$	$\overline{\epsilon}$	$\overline{\epsilon}$	$\overline{\epsilon}$	$\overline{\epsilon}$	All
2.	$(\overline{\epsilon}, \overline{\epsilon}, \underline{\epsilon})$	$\frac{\overline{\epsilon}}{3}$	$\overline{\epsilon}$	$0.5\overline{\epsilon} + 2D\overline{\epsilon}$	$-0.5\overline{\epsilon} - 2D\overline{\epsilon}$	C.
3.	$(\overline{\epsilon}, \underline{\epsilon}, \overline{\epsilon})$	$\frac{\overline{\epsilon}}{3}$	$0.5\overline{\epsilon} - 2D\overline{\epsilon}$	0	$+0.5\overline{\epsilon} + 2D\overline{\epsilon}$	E.,W.
4.	$(\overline{\epsilon}, \underline{\epsilon}, \underline{\epsilon})$	$-\frac{\overline{\epsilon}}{3}$	$0.5\overline{\epsilon} - 2D\overline{\epsilon}$	$-0.5\overline{\epsilon} + 2D\overline{\epsilon}$	$-\overline{\epsilon}$	C.
5.	$(\underline{\epsilon}, \overline{\epsilon}, \overline{\epsilon})$	$\frac{\overline{\epsilon}}{3}$	$-0.5\overline{\epsilon} + 2D\overline{\epsilon}$	$0.5\overline{\epsilon} - 2D\overline{\epsilon}$	$\overline{\epsilon}$	C.
6.	$(\underline{\epsilon}, \overline{\epsilon}, \underline{\epsilon})$	$-\frac{\overline{\epsilon}}{3}$	$-0.5\overline{\epsilon} + 2D\overline{\epsilon}$	0	$-0.5\overline{\epsilon} - 2D\overline{\epsilon}$	E.,W.
7.	$(\underline{\epsilon}, \underline{\epsilon}, \overline{\epsilon})$	$\frac{-\overline{\epsilon}}{3}$	$-\overline{\epsilon}$	$-0.5\overline{\epsilon} - 2D\overline{\epsilon}$	$0.5\overline{\epsilon} + 2D\overline{\epsilon}$	C.
8.	$(\underline{\epsilon}, \underline{\epsilon}, \underline{\epsilon})$	$-\overline{\epsilon}$	-$\overline{\epsilon}$	- $\overline{\epsilon}$	- $\overline{\epsilon}$	All

Table 2.2: The Impact of Asymmetry between Peripheral Countries on Monetary Policy

We need now to determine whether the expected loss function for the East is higher under Majority Voting or Secret Voting. To this end, we can exploit an analogously expression to (2.2.44):

$$E\left[L_e(\pi^{tv,*}, z_e, D) - L_e(\pi^{sv,*}, z_c, \overline{z}, D)\right] = E\left\{\frac{\partial^2 L_e(\pi^{*,e})}{\partial \pi}\left[\left(\pi^{*,e} - \pi^{tv,*}\right)^2 - \left(\pi^{*,e} - \pi^{sv,*}\right)^2\right]\right\}; \tag{2.2.58}$$

To derive equation (2.2.58), we go through a similar procedure employed to derive (2.2.44). We write a second order Taylor expansion for $L_e(\pi^{tv,*}, z_e)$ and $L_e(\pi^{sv,*}, \overline{z})$ around the point $\pi^{*,e}$, where $\pi^{*,e}$ denotes the optimal choice of the inflation rate the East would have carried out if it conducted an independent monetary policy. We then exploit the first order condition that $L'(\pi = \pi^{*,e}) = 0$, take expectations for both expressions and subtract the expression for $L_e(\pi^{sv,*}, \overline{z})$ from the Taylor expansion for $L_e(\pi^{tv,*}, z_e)$.

We now use the results of table (2.2) to calculate the value of the terms on the right hand side of (2.2.58). The format in this table is analogous to the one in Table 2.1.

Table 2.2 describes the outcome of monetary policy when output supply shocks take the form stated in (2.2.56). The second column details the nature of the supply shock occurring to each industry, which are averaged in the third column; columns four to six describe precisely the output supply shock occurring in each region, which is derived using (2.2.56). The last column specifies which country acts as the median voter in each case.

Using (2.2) we find that (recalling that z^{mv} describes the output supply shock occurring to the median voter):

$$E\left[z_e - z^{mv}\right]^2 = \frac{1}{4}\sigma_\epsilon^2\left[20D^2 + 6D - \frac{5}{4}\right]; \tag{2.2.59}$$

We can similarly compute:

$$E\left[z_e - \overline{z}\right]^2 = \frac{1}{4}\sigma_\epsilon^2\left[(+8D^2 - 4D + \frac{7}{6}\right]; \tag{2.2.60}$$

Bearing in mind that equations (2.2.11), (2.2.25) and (2.2.19) imply that:

$$E\Big[\pi^{*,e} - \pi^{sv,*} = \Big]^2 = (\beta+\gamma)^{-2}E\Big[\overline{z} - z^{*e}\Big]^2; \tag{2.2.61}$$

$$E\Big[\pi^{*,e} - \pi^{tv,*} = \Big]^2 = (\beta+\gamma)^{-2}E\Big[z^{mv} - z_e\Big]^2; \tag{2.2.62}$$

Substituting this back into equation (2.2.58) and noticing again that (2.2.1), (2.2.2) and (2.2.10) imply that $L''(\pi) = 2(\beta+\gamma)$, we can determine what voting regime is welfare optimal for the East:

$$E\Big[L_e(\pi^{tv,*}, z_e, \overline{z}, D) - L_e(\pi^{sv,*}, z_e, \overline{z}, D)\Big] = \frac{1}{4}\sigma_\epsilon^2\frac{\left(+\frac{19}{12} - 10D - 12D^2\right)}{[2(\beta+\gamma)]}\begin{cases} > 0 & if\ D < 0.136; \\ = 0 & if\ D = 0.136; \\ < 0 & if\ D > 0.136; \end{cases} \tag{2.2.63}$$

Hence if D is sufficiently high (that is if the East is sufficiently similar to the Center which in most cases acts as the median voter) then $E\Big[L_e(\pi^{sv,*}, z_e, \overline{z}, D) - L_e(\pi^{tv,*}, z_e, \overline{z}, D)\Big] > 0$ so that Transparency Voting is welfare superior for the East.

We now need to go through a similar process to determine which Transparency Voting Regime is optimal for the Center. We therefore use Table 2.2 to compute:

$$E(z^{*,c} - z^{mv})^2 = \frac{1}{4}\sigma_\epsilon^2\left[(-\frac{1}{2} + 2D)^2\right]; \tag{2.2.64}$$

$$E(z^{*c} - \overline{z})^2 = \frac{1}{4}\sigma_\epsilon^2\left[\frac{1}{6} + 8D_1^2\right];$$

Finally, substituting (2.2.64) into (2.2.48) and into (2.2.46) and then using equation (2.2.44) we obtain:

$$E\Big[L_c(\pi^{tv,*}, z_c, \overline{z}, D) - L_c(\pi^{sv,*}, z_c, \overline{z}, D)\Big] = \sigma_\epsilon^2 \frac{\left(+\frac{1}{48} - 0.5D - 4D^2\right)}{2(\beta+\gamma)} \begin{cases} > 0 & if\ D < 0.038; \\ = 0 & if\ D = 0.038; \\ < 0 & if\ D > 0.038; \end{cases} \tag{2.2.65}$$

The Center is therefore better off with Transparent Voting if and only if $D > 0.038$. Therefore, we notice that whenever Transparent Voting Rules are optimal for the East, they are also optimal for the Center. The only incentive compatibility condition that is binding is the one for the East. Thus, by checking (2.2.63) we conclude that whenever $D > 0.136$, a majority of countries in the Monetary Union are better off with Transparent Voting Rules, even though voting secrecy is welfare optimal for the Monetary Union as a whole. □

The intuition for the result is best understood considering the polar case in which $D = \frac{1}{4}$. Then the East and the Center have perfectly aligned voting incentives since they experience identical output supply shocks. In this polar case, if monetary policy is conducted by Voting Transparency the East and the Center both act as the median voter in all contingencies and are therefore able to obtain their first best choice of monetary policy in all cases. This explains why if D is sufficiently high, the East and the Center favor Transparency Voting.

On the other hand, if D is sufficiently low, the Center and the East cannot be certain that they shall act as the median voter in all cases, since their output supply shocks are not identical, though they might be similar. In some contingencies, for instance, the Center will be out-voted by the East and the West, and therefore it might prefer that monetary policy be conducted via Voting Secrecy, which acts, as previously discussed, as an insurance policy against the risk of being sharply out-voted. In fact, under Voting Secrecy all countries know that their preferences shall be at least partially taken into account in every contingency.

Conclusively, note also that the result of the first part of Proposition 2.2.1, stating that Secret Voting is welfare optimal for the Monetary Union as a whole, does not rest upon the choice of a particular functional form for output supply shocks. This is so for the result just follows from the assumption that monetary policy is conducted by a Committee of benevolent social planners if Voting Secrecy removes the incentive to vote according to partisan interests. Therefore, the East is made worse off by Transparent Voting even if the East and the Center may be better off under this rule. As a result, regardless of the magnitude of D, Secret Voting is welfare optimal for the Monetary Union as a whole as long as we can take the ECB's assumption at face value.

This observation concludes this section. We now introduce a different analytical framework to study the effects of letting the geographic pattern of industrial structure be endogenous to the monetary policy.

2.3 Endogeneity in Industrial Location and Voting Secrecy

The results developed in the previous section rest on the assumption that industrial location is *exogenous* to the decision of which Monetary Policy voting rule to adopt in a Monetary Union.

We now aim to investigate the link between the firm's decision about industrial location and the choice of Monetary Policy Voting Rule. Would the result that Voting Secrecy, as opposed to Transparent Voting, is welfare optimal still persist when industrial location is endogenous to monetary policy?

We study the decision of industrial location inside a Monetary Union through a general equilibrium framework in which the representative agent/firm has the choice of either locating its productive activities in all the three regions of the Monetary Union (*thus hedging against macroeconomic fluctuations occurring at the region-wide level*) or can instead focus on producing in a single region (*thus locating production in a region that enjoys a comparative advantage for the production of a given good*).

The overall aim in this section lies in showing that Transparent Voting can induce *each industry to locate more widely across all the regions of the Monetary Union* as opposed to producing from a single location. Therefore, though we here study a model in which all the stochastic shocks are demand ones, our results indicate that Transparent Voting can *lessen the asymmetry of supply shocks by inducing firms not to geographically specialize their productive activities.* In fact, the industrial structure across member country becomes more similar as firms in a given industry choose to locate their activities widely at the Monetary-Union wide level. This would render supply shocks more symmetric.

It might be useful to preview the intuition of the model delivering the above result: why are firms induced not to geographically specialize? As transportation costs, dishomogeneity in the preference for some good characteristic and similar factors imply that an important share of demand tends to be concentrated in the region where output is produced, firms become heavily exposed to local macroeconomic fluctuations when they firms specialize production (or sourcing) in a given region.

To diversify production implies that firms are to some degree able to hedge against idiosyncratic demand shocks hitting a certain region. We can therefore analyze the decision or whether to locate in all regions or just in one as being similar to a portfolio allocation decision, in which the incentive to increase portfolio diversity is rising in the volatility that the investor would face if she did not hedge her portfolio.

How does the choice of of which monetary policy voting rule to adopt affect firms decisions as to whether to locate in only one or in all the three regions of the Monetary Union? To shed light on this question, it is worth carrying a little further the analogy between the choice of industrial location and portfolio diversification.

Transparent Voting implies that, as we shall show, aggregate demand in each region is more volatile than under Secret Voting (for if a country gets out-voted, no effort is made under Transparency Voting rules to take its preferences into account when monetary policy is set). Therefore locating in the single region where a given industry enjoys a comparative advantage (*that is, holding a unhedged portfolio that puts a full weight on the asset endowed with the highest return*) exposes the firm to greater risk under Transparent Voting that under Secret Voting. For this reason, firms are more likely to spread themselves across all the three regions under Transparent Voting.

However, if firms spread across all the three regions, then all regions face the same industrial structure, so that supply shocks would have the tendency to become symmetric across the Monetary Union.

We now seek to develop a simple general equilibrium model to formalise these qualitative insights.

2.3.1 A General Equilibrium Framework

We now carry out the analysis via a simple general equilibrium framework which is essentially an extension of a model *á la* Blanchard and Kiyotaki (Blanchard and N.Kiyotaki 1987). Three islands compose our modeled Monetary Union. We denote islands e,c,w with subscript m, with m=1,2,3.

2.3.1.1 The Structure of the Game:

The monetary policy game can be divided into the following stages:

1. The Central Bank of the Monetary Union announces whether it shall conduct monetary policy by Transparent Voting Rules or by Secret Voting; we assume again that under Transparent Voting all members of the monetary policy setting body vote strictly by partisan interests; on the other hand, under Secret Voting all members of the monetary setting authority are freed from partisan pressure and can therefore vote as if they were benevolent social planners taking into account the preferences of all the regions composing the Monetary Union.

2. Agents, each of which acts as the monopolist producer for three goods in the same given industry, face the choice of either locating in one island, from which they produce all the three goods they are a monopolist for, or spreading their production, their labor activities and their consumption widely across the three islands of the Monetary Union. Each agents decides on whether to localize in one or three locations, but we restrict the analysis to symmetric equilibria only, in which all agents (who are ex-ante identical) choose the same strategy. We show in Proposition 2.3.1 that there always exists a symmetric equilibrium in which either all agents choose to locate only in one island or all agents choose to spread their activities widely across the all Monetary Union.

3. Idiosyncratic shocks to money supply in each island are realized. The Monetary authority then decides to stabilize the money supply via monetary policy, though it faces a restriction: it can contract or restrict the money supply as it wishes, subject to the constraint that *changes in the money supply must be the same across all islands* . This

assumption reflects the fact that all member countries of a Monetary Union are subject to a common monetary policy.

4. After learning the value of the money supply with full and complete information, the economy determines its equilibrium values of labor and consumption in each island. Fluctuations in Money Supply feed upon macroeconomic fundamentals as we assume that wages are sticky.

2.3.1.2 Agents Utility Function and Budget Constraint in a Three Islands Model:

The representative agent i draws utility from the sum of each consumption basket she carries out in each island $(C_{i,m})$ and from the sum of her real money holdings in each island $(\frac{M_{i,m}}{P_i})$ at a diminishing rate over scale, while disutility is derived from labour N_i at an increasing rate over scale, so that the utility function, subject to a dummy variable D_i whose interpretations we discuss below, takes the form:

$$u_i = \left(\frac{\sum_{m=1}^{3} C_{i,m}}{d}\right)^d \left(\frac{\sum_{m=1}^{3}\left(\frac{M_{i,m}}{P_m}\right)}{1-d}\right)^{1-d} - \left(\frac{\sum_{m=1}^{3} N_{i,m}}{b}\right)^b - D_i\tau; \tag{2.3.1}$$

$$1 > d > 0; b > 1;$$

We now proceed to illustrate the meaning of the dummy variable D_i by formulating the following assumption:

Assumption 2.3.1. ***(Cost of Diversifying Geographic Localization):*** *The economy is divided into three groups of agents of equal size (three industries), each of which has a comparative advantage in locating its productive and consumption activities in one of the three regions of the Monetary Union. Goods 1 to r belong to industry e, endowed with a comparative advantage in the eastern region. Goods r+1 to 2r belong to the industry enjoying a comparative advantage in the Central Region, and goods 2r+1 to 3r belong to the industry enjoying a comparative advantage in the Western Region.*

There are r agents, each acting as the monopolist producer for three goods in the same industry.

The variable τ can be interpreted as the cost of locating production away from the region where a given agent enjoys her comparative advantage.

Therefore, D_i takes the value of zero if the agent decides to specialize in consuming and producing only in the single island in which the productive activities of the industry in which she operates enjoy a comparative advantage.

Alternatively, the agent can choose to produce and consume in all the three islands, implying that $D_i = 1$ and therefore she will have to pay the cost τ of locating her activities in all islands of the Monetary Union.

We denote with r_m the number of goods produced in each island. As we show in Proposition 2.3.1, there exists a symmetric equilibrium in which r goods are produced in each island.

The consumption index in each island is determined by:

$$C_{i,m} = r_m^{\frac{1}{1-\sigma}} \Big(\sum_{j=1}^{r_m} C_{j,m}^{\frac{\sigma-1}{\sigma}} \Big)^{\frac{\sigma}{\sigma-1}}, \sigma > 1; \tag{2.3.2}$$

Consumers display no love for consumption variety, as we can show by noticing that the following associated price index is not falling in r_m if all individual prices are the same:

$$P_m = \left(\frac{1}{r_m} \sum_{j=1}^{r_m} p_j^{(1-\sigma)} \right)^{(\frac{1}{1-\sigma})} ; \tag{2.3.3}$$

Islands do not trade and consumers are allowed to spend their income only in the region in which they have earned it. Therefore, income must equal expenditure in each island. Denoting profits accruing in island m to agent i with $\pi_{i,m}$, the initial money endowment as M_m^0 and the wage rate as w_m, we can write consumers' budget constraint as:

$$\left(\sum_{j=1}^{r_m} p_{m,j} c_{m,j,i} + M_{i,m} \right) = \left(\overline{W_m} N_{i,m} + \pi_{i,m} + M_{i,m}^0 \right) = (\Omega_{i,m}) \;\; \forall i, m; \tag{2.3.4}$$

Though restricting trade among islands is not realistic (presumably countries join a Monetary Union precisely because they trade heavily with each other and we have here assumed that all goods are non-tradeable), not modeling explicitly trading links greatly simplifies the analysis.

2.3.1.3 Optimal Choice of Consumption and Real Money Holdings:

We employ the well known two-stages budgeting technique to compute the optimal demand choices of agents in each island. In the first stage, the consumer decides how much consumption to carry out in each island and how much money holding to hold. In the second stage, it actually decides how to allocate its consumption across the various product varieties. We then first set up the Lagrangean multiplier to maximize the utility function of (2.3.1) subject to (2.3.4):

$$L_{1,i} = \left(\frac{\sum_{m=1}^{3} C_{i,m}}{d}\right)^{d} \left(\frac{\sum_{m=1}^{3}\left(\frac{M_{i,m}}{P_m}\right)}{1-d}\right)^{1-d} - \left(\frac{\sum_{m=1}^{3} N_{i,m}}{b}\right)^{b} - D_i\tau + \\ + \sum_{m}^{3} \lambda_{i,m}\left[P_m C_{i,m} + M_{i,m} - w_m n_{i,m} - \pi_{i,m} - M_{i,m}^{0}\right]; \tag{2.3.5}$$

This yields first order conditions:

$$C_{i,m} : d\left(\frac{\sum_{m=1}^{3} C_{i,m}}{d}\right)^{d-1} \left(\sum_{m=1}^{3} \frac{\frac{M_{i,m}}{P_{i,m}}}{1-d}\right)^{1-d} = -\lambda_m P_m; \tag{2.3.6}$$

$$M_{i,m} : (1-d)\left(\sum_{m=1}^{3} \frac{C_{i,m}}{d}\right)^{d} \left(\sum_{m=1}^{3} \frac{\frac{M_{i,m}}{P_{i,m}}}{1-d}\right)^{-d} = -\lambda_m P_m; \tag{2.3.7}$$

$$\lambda_{i,m} : \left[P_m C_{i,m} + M_{i,m} - w_m n_{i,m} - \pi_{i,m} - M_{i,m}^{0}\right] = 0; \tag{2.3.8}$$

We explain in Section 2.3.1.4 why we do not derive a first order condition for labor supply.

Dividing equation (2.3.6) by (2.3.7) first order conditions turn out to imply:

$$\frac{d}{1-d}\sum_{m=1}^{m=3}\left(\frac{M_{i,m}}{P_m}\right) = \sum_{m=1}^{m=3}\left(C_{i,m}\right); \tag{2.3.9}$$

The solution to (2.3.5) turns out to be the standard Cobb-Douglas result:

$$\frac{M^*_{i,m}}{P^*_m} = (1-d)\frac{\Omega_{i,m}}{P_m}; \qquad (2.3.10)$$
$$C^*_{i,m} = d\frac{\Omega_{i,m}}{P_m};$$

To verify that (2.3.10) is indeed a solution to (2.3.5), we substitute appropriately (2.3.10) into (2.3.9), which confirms that both sides of the first condition of (2.3.9) would then be equal to $\sum_{m=1}^{3}\frac{\Omega_m}{P_m}$. This then verifies the fact that (2.3.10) solves (2.3.5).

2.3.1.4 The Labor Market

Were the wage to be flexible and was the labor market to clear, then we would need to derive a first order condition that links labor supply to the real wage rate. However, we assume that the sticky wage $\overline{w_m}$ is always above the wage that would clear the labor market, so that there is excess supply of labor. And the quantity of labor demanded is accommodated by labor supply at the prevailing wage $\overline{w_m}$.

The assumption of a sticky wage drives the later result that money is non-neutral and that therefore demand shocks bring about fluctuations in macroeconomic variables. In fact, without the assumption of wage stickiness and absent price adjustment costs, the usual result of money neutrality would hold.

2.3.1.5 General Equilibrium in Each Island:

We now turn attention to the determination of General Equilibrium in each island.

The Demand Side:

Using equation (2.3.10) we can establish the following relationships between consumption and real money holdings in each island:

$$C^*_{i,m} = \frac{d}{1-d}\frac{M^*_{i,m}}{P_m}; \qquad (2.3.11)$$
$$C^*_m = \frac{d}{1-d}\frac{M^*_m}{P_m}; \qquad (2.3.12)$$

The second equation is derived by aggregate across all agents in each island. We can use two stages budgeting to derive the demand for each good j in each island so that the consumer in each island decides how to allocate her expenditure $P_{i,m}C_{i,m}$ across the j goods as to maximize:

$$L_{2,i} = r^{\frac{1}{1-\sigma}} \left(\sum_{j=1}^{r_m} c_{i,j}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \lambda_2 \left(\sum_{j=1}^{r_m} P_{j,m} C_{i,j,m} - P_m C_{i,m} \right); \tag{2.3.13}$$

The solution of the problem, aggregating over all consumers in a island, yields:

$$C^*_{j,m} = \left(\frac{P_{j,m}}{P_m} \right)^{-\sigma} \left(\frac{d}{1-d} \right) \frac{M^*_m}{r_m P_m} \quad \forall j = 1, .., r_m; \tag{2.3.14}$$

Having determined the demand side, we not turn attention to aggregate supply.

The Supply Side:

Each firm in each island must choose the price ($P_{j,m}$) and the quantity of labor it wishes to employ with the view of maximizing profits:

$$max\ \pi_{j,m} = P_{j,m} Y_{j,m} - \overline{w} N_{j,m}; \tag{2.3.15}$$

Profit maximization is subject to two constraints. The first constraint dictates that, after that each firm sets its price, the quantity is driven by the demand function derived in (2.3.14), so that each firm faces the following demand curve:

$$Y_{j,m} = \left(\frac{P_{j,m}}{P_m} \right)^{-\sigma} \left(\frac{d}{1-d} \right) \frac{M^*_m}{r_m P_m}\ \forall j = 1, .., r_m; \tag{2.3.16}$$

The second constraint dictates that labor is subject to diminishing marginal returns:

$$Y_{j,m} = (N_{j,m})^{\frac{1}{\alpha}}, \alpha > 1; \tag{2.3.17}$$

Equilibrium demand in the typical firm is therefore, after maximizing the profit function of (2.3.15) subject to (2.3.16) and (2.3.17):

$$Y^*_{j,m} = \left(\alpha \frac{\overline{w} Y_m^{\alpha-1}}{(1-\mu) P_m} \right)^{-\sigma} \left(\frac{d}{1-d} \right) \frac{M^*_m}{r_m P_m}; \tag{2.3.18}$$

where:

$$\mu = \frac{P_j - \alpha\overline{w}(Y_{j,m})^{\alpha-1}}{P_j}; \tag{2.3.19}$$

General Equilibrium Solution:

We now study the characterization of a symmetric equilibrium in the product market, one in which all firms belonging to the same island choose the same price, output and level of employment. Hence, symmetry implies that $P_{j,m} = P_{-j,m}$.

Using the price index in (2.3.3), it follows that:

$$P^*_m = \left[\frac{1}{r_m} r_m (P^*_{j,m})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}; \tag{2.3.20}$$

Solving (2.3.18) for Y_m, after aggregating over r_m and exploiting the implication of (2.3.20) that $P_m = P_{j,m}$, yields the following expression for aggregate output in each island:

$$Y^*_m = \frac{d}{1-d}\frac{M^*_m}{P^*_m}; \tag{2.3.21}$$

Furthermore, we employ the constraint of equation (2.3.17) to determine employment:

$$N^*_m = \sum_j^{r_m} N_{j,m} = (Y^*_m)^{\alpha}; \tag{2.3.22}$$

Also notice that real aggregate demand is proportional to real money holdings. To see that, notice that first of all money market equilibrium implies that $M^*_m = M^0_m$. Substituting the money market equilibrium into (2.3.11) one gets:

$$C^*_m = \frac{d}{1-d}\frac{M^0_m}{P^*_m}; \tag{2.3.23}$$

Our final task lies in determining the price level in each island as to close the model. For markets to clear, the quantity supplied for each good j $Y^s_{j,m}$ and the quantity demanded $C_{j,m}$ must be equal in each island. Therefore, equating (2.3.14) with (2.3.18), we verify that goods market clear only if the following condition is verified:

$$log\left(\frac{\overline{w}}{P^*_m}\right) = -(\alpha-1)log\left(\frac{M^0_m}{P_m}\right) + log(1-\mu) + log(\alpha) - (\alpha-1)log(\frac{c}{1-c}); \tag{2.3.24}$$

This finally implies that the price level in each island for the goods market to clear must be:

$$log(P_m^*) = \left(\frac{log(\overline{w} + (\alpha - 1)log(M_m^0) - k_1)}{\alpha}\right); \qquad (2.3.25)$$

where:

$$k_1 = log(1-\mu) + log(\alpha) - (\alpha - 1)log(\frac{c}{1-c}); \qquad (2.3.26)$$

We now turn attention to the determination of monetary policy.

2.3.2 Monetary Policy Voting Rules and the Location of Industry

This section aims to compare the conduct of monetary policy under Transparent Voting and under Secret Voting. But before proceeding to this task, it might be useful to briefly fix ideas on why monetary policy plays any role at all in determining output, labor and consumption.

2.3.2.1 Fluctuations in Aggregate Demand and the Role of Monetary Policy:

Absent menu costs a general equilibrium model of the kind we have here developed would produce money neutrality: monetary policy would not effect real variables. In fact, in a general equilibrium model without wage stickiness the price level is homogenous of degree one in money supply, so that the quantity of money cannot affect the ratio of money over prices and the real wage.

However, the assumption we have formulated about wage rigidity implies that an increase in aggregate demand lowers the real wage. In fact, a rise in aggregate demand acts to increase prices while the nominal wage stays constant. From this mechanism stems the link between fluctuations in aggregate demand and fluctuations in real macroeconomic variables.

We posit that aggregate demand fluctuations arise from the very *fluctuation of the supply of monetary aggregates* M_m^0, which we break down into two components, to be defined and interpreted below:

$$M_m^0 = \overline{M_m^0} + \overline{\Delta M_m^0}; \tag{2.3.27}$$

The first component $\overline{M_m^0}$ of money supply in each island is a stochastic term that captures the value that the money supply would take in each island *if monetary policy were neutral.*

$\overline{M_m^0}$ fluctuates around its expectation $\overline{M}$ because of binomial idiosyncratic shocks to money supply. In half of the cases, such random component takes on a high value, and in half of the cases it takes on a low value, so that:

$$\overline{M_m^0} = \begin{cases} \overline{M} + \overline{\epsilon} & = M^H \quad with\ Pr\frac{1}{2}; \\ \overline{M} - \overline{\epsilon} & = M^L \quad with\ Pr\frac{1}{2}; \end{cases} \tag{2.3.28}$$

The second component $\overline{\Delta M_m^0}$ of M_m^0 in (2.3.27) captures, instead, the effect of monetary policy on the money supply. If $\overline{\Delta M_m^0} > 0$, monetary policy is expansive. Otherwise, monetary policy is restrictive.

What are the implications of assuming that the constraints of a Monetary Union apply for the determination of the Money Supply? We detail this in the next assumption.

Assumption 2.3.2. ***(Monetary Union Restrictions):*** *A common monetary policy implies that monetary policy must increase or reduce the money supply by the same magnitude across all the three islands m=1,2,3. This translates into the following restriction:*

$$\overline{\Delta M_1^0} = \overline{\Delta M_2^0} = \overline{\Delta M_3^0}; \tag{2.3.29}$$

Against the background of equation (2.3.27) and of the restriction outlined in Assumption 2.3.2 operates monetary policy.

2.3.2.2 The Conduct of Monetary Policy:

Monetary Policy is decided by the Board of the Monetary Union Central Bank. Each island holds one seat in such committee. To understand how monetary policy is conducted in the Monetary Union it might be useful to ask how would each island operate if it were to conduct monetary policy in an independent fashion.

We assume that under independence monetary policy in each island would aim to minimize the following loss function:

$$L_m = \left(M_m^0 - \overline{M}\right)^2; \tag{2.3.30}$$

Why would each island under independence aim to stabilize fluctuations in the money supply? It might be observed that the general equilibrium model presented in the previous section encompasses no welfare loss attached to instability in the price level. Furthermore, the assumption of imperfect competition implies that in equilibrium output is below the welfare optimal level.

Admittingly, the assumption of 2.3.30 cannot be supported by the micro-foundations of the model. However, it might be conjectured that if the Central Bank did not try to peg the money supply to a certain value there might be no factor holding the money supply from growing at an indefinite rate (for absent a pegging mechanism for the money supply the Central Bank could always be tempted to increase $\frac{m}{p}$ as to take output above its Walrasian sub-optimal equilibrum, regardless of the price level. Then, faced with an increase in the money supply, agents would have no choice but to rise prices). This might offer a rational to having a monetary target even if the general equilibrium framework in which we operate.

Would there be any fluctuation in aggregate demand were the three islands to be able to conduct monetary policy independently? If the restriction of (2.3.29) does not hold, each island would be able to achieve any level of money supply it wishes by setting $\overline{\Delta M_m^0} = -\left(\overline{M} - \overline{M_m^0}\right)$.

However, the existence of a Monetary Union imply that money supply might actually fluctuate around its target as monetary policy in each island faces an external constraint. But the extent upon which the money supply fluctuates around its target varies according to the choice of voting rule, as we now set to show.

Monetary Policy Under Transparent Voting Rules:

We assume that under Transparent Voting all members of the Voting Panel are forced to vote according to their own partisan interests.

Hence under Transparent Voting rules, the representative of each island votes as to minimizes the loss function of (2.3.30), so that the preferred monetary stance for the representative of each island is equal to:

$$\overline{\Delta M_m^{tv,0}} = \overline{M} - \overline{M_m^0} \tag{2.3.31}$$

Under Transparency Voting Rules, the preference of the median voter gets implemented, so that each island if acting as the median voter manages to ensure that the value for the money supply realized after monetary policy is implemented is equal to the target level $\overline{M}$. Therefore, by denoting with $\overline{\Delta M^{tv,mv}}$ the preference for monetary policy of the country that acts as the median voter determined by (2.3.31) (that is, of the island that gets hit by the median value of the shock to $\overline{M_m^0}$) the money supply in each island is equal to:

$$M_m^{tv,0} = \overline{M}_m^0 + \overline{\Delta M^{tv,mv}}; \tag{2.3.32}$$

The outcome of monetary policy in each contingency is summarized in Table 2.3.

The first column of the table depicts the value for the money supply $\overline{M_m^0}$ in each island *that would hold if monetary policy stayed neutral.* This is denoted with M^H if a positive shock to the money supply has initially occurred. Instead, the first entry in the column reads M^L if a negative supply to the money shock has occurred, so that, absent active monetary policy, the quantity of money in circulation would be curtailed.

The second column of the table describes values taken by the money supply in each island after the monetary policy move is implemented. To calculate this, one must add the value taken by $\overline{\Delta M^{tv,mv}}$ (the change in the money supply induced by active monetary policy) to the entry for each island recorded in first column of the table (depicting the value for the money supply that would obtain absent active monetary policy). So, for example, if the reading for the first column is M^H for a given country, so that a positive money supply shock of magnitude $\overline{\epsilon}$ has taken place, the reading for the resulting quantity of money $M^{*,0}$ in each island in the second column would be $\overline{M}$ if $\overline{\Delta M^{tv,mv}} = -\overline{\epsilon}$ so that the median voter has decided to, loosely speaking, withdraw from circulation the same quantity of money created by the exogenous monetary and aggregate demand shock that has occurred in the median's voter island. The net result would be, therefore, that in this case the quantity of money in circulation after monetary policy is implemented is $\overline{M}$.

$\left(\overline{M_1^0}, \overline{M_2^0}, \overline{M_3^0}\right)$	$\left(M_1^{*,0}, M_2^{*,0}, M_3^{*,0}\right)$	$\Delta\overline{M^{*,0}}$
1. $\left(M^H, M^H, M^H\right)$	$\left(\overline{M}, \overline{M}, \overline{M}\right)$	$-\overline{\epsilon}$
2. $\left(M^H, M^H, M^L\right)$	$\left(\overline{M}, \overline{M}, \overline{M} - 2\overline{\epsilon}\right)$	- $\overline{\epsilon}$
3. $\left(M^H, M^L, M^H\right)$	$\left(\overline{M}, \overline{M} - 2\overline{\epsilon}, \overline{M}\right)$	- $\overline{\epsilon}$
4. $\left(M^H, M^L, M^L\right)$	$\left(\overline{M} + 2\overline{\epsilon}, \overline{M}, \overline{M}\right)$	$+\overline{\epsilon}$
5. $\left(M^L, M^H, M^H\right)$	$\left(\overline{M} - 2\overline{\epsilon}, \overline{M}, \overline{M}\right)$	$-\overline{\epsilon}$
6. $\left(M^L, M^H, M^L\right)$	$\left(\overline{M}, \overline{M} + 2\overline{\epsilon}, \overline{M}\right)$	$+\overline{\epsilon}$
7. $\left(M^L, M^L, M^H\right)$	$\left(\overline{M}, \overline{M}, \overline{M} + 2\overline{\epsilon}\right)$	$+\overline{\epsilon}$
8. $\left(M^L, M^L, M^L\right)$	$\left(\overline{M}, \overline{M}, \overline{M}\right)$	$+\overline{\epsilon}$

Table 2.3: The Impact of Transparency Voting on Monetary Policy

The third column depicts the stance of monetary policy $\overline{\Delta M^{*,0}}$ which is eventually implemented. This is set equal to $\overline{\Delta M^{tv,mv}}$, the preference of the median voter.

We employ the table to calculate the variance of the money supply in each island, which we record for future reference:

$$VAR\left(M_m^{tv,0}\right) = \left(\overline{\epsilon}\right)^2; \tag{2.3.33}$$

Monetary Policy Under Secret Voting:

As previously assumed, under Secret Voting the Monetary Policy Committee aims, subject to the restriction that $\overline{\Delta M_m^0}$ must be the same across countries as assumed in (2.3.2), to minimize the following Pan-Union loss function:

$$L^{sv} = \sum_{m=1}^{m=3} \left(\overline{M_m^0} - \overline{M}\right)^2; \tag{2.3.34}$$

It must be stressed again that Secret Voting follows a benevolent social planner rule since all members of the interest rate voting panel are free from partisan interests because their individual voting records are kept secret. Hence, all members of the interest voting panel, being free from partisan pressure, just try to achieve the Union-wide money supply stabilization target. This results into the voting rule:

$$\overline{\Delta M^{sv,o}} = \frac{\left(\sum_{m=1}^{m=3}(\overline{M} - \overline{M_m^0})\right)}{3}; \tag{2.3.35}$$

$\left(\overline{M_1^0}, \overline{M_2^0}, \overline{M_3^0}\right)$	$\left(M_1^{*,0}, M_2^{*,0}, M_3^{*,0}\right)$	$\Delta\overline{M^{*,0}}$
1. $\left(M^H, M^H, M^H\right)$	$\left(\overline{M}, \overline{M}, \overline{M}\right)$	$-\overline{\epsilon}$
2. $\left(M^H, M^H, M^L\right)$	$\left(\overline{M} + \frac{2}{3}\overline{\epsilon}, \overline{M} + \frac{2}{3}\overline{\epsilon}, \overline{M} - \frac{4}{3}\overline{\epsilon}\right)$	$-\frac{1}{3}\overline{\epsilon}$
3. $\left(M^H, M^L, M^H\right)$	$\left(\overline{M} + \frac{2}{3}, \overline{M} - \frac{4}{3}\overline{\epsilon}, \overline{M} + \frac{2}{3}\overline{\epsilon}\right)$	$-\frac{1}{3}\overline{\epsilon}$
4. $\left(M^H, M^L, M^L\right)$	$\left(\overline{M} + \frac{4}{3}\overline{\epsilon}, \overline{M} - \frac{2}{3}\overline{\epsilon}, \overline{M} - \frac{2}{3}\overline{\epsilon}\right)$	$+\frac{\overline{\epsilon}}{3}$
5. $\left(M^L, M^H, M^H\right)$	$\left(\overline{M} - \frac{4}{3}\overline{\epsilon}, \overline{M} + \frac{2}{3}\overline{\epsilon}, \overline{M} + \frac{2}{3}\overline{\epsilon}\right)$	$+\frac{1}{3}\overline{\epsilon}$
6. $\left(M^L, M^H, M^L\right)$	$\left(\overline{M} - \frac{2}{3}\overline{\epsilon}, \overline{M} + \frac{4}{3}\overline{\epsilon}, \overline{M} - \frac{2}{3}\overline{\epsilon}\right)$	$+\ \frac{1}{3}\overline{\epsilon}$
7. $\left(M^L, M^L, M^H\right)$	$\left(\overline{M} - \frac{2}{3}\overline{\epsilon}, \overline{M} - \frac{2}{3}\overline{\epsilon}, \overline{M} + \frac{4}{3}\overline{\epsilon}\right)$	$-\frac{1}{3}\overline{\epsilon}$
8. $\left(M^L, M^L, M^L\right)$	$\left(\overline{M}, \overline{M}, \overline{M}\right)$	$+\overline{\epsilon}$

Table 2.4: The Impact of Secret Voting on Monetary Policy

As a result of this voting rule, the money supply in each island takes the following form:

$$M_m^{sv,0} = \overline{M_m^0} + \frac{\sum_{m=1}^{m=3}\left(\overline{M} - \overline{M_m^0}\right)}{3}; \tag{2.3.36}$$

The behavior of monetary policy under this voting rule is summarized by Table 2.4. Every column has the same interpretation as in the previous table, except for the third column, which now describes the unanimous choice of monetary policy taken by the Committee, rather than the decision imposed by the median voter as in the previous scenario.

We make use of the results of the table to compute the variance of the money supply in each island under Voting Secrecy:

$$VAR\left(M_m^{sv,0}\right) = \frac{2}{3}\left(\overline{\epsilon}\right)^2; \tag{2.3.37}$$

A comparison of the Impact of Voting Rules across Regimes:

It is useful at this stage to compare the outcome of monetary policy across the two regimes by referring to Table 2.3 and Table 2.4. First of all, notice that in contingencies 1. and 8. the outcome of the two rules does not differ. In these cases, all the three islands are hit by an output supply shock of the same magnitude and therefore even under Transparency Voting no disagreement arises among the policy-makers about the optimal monetary stance.

The two rules have a different impact in the remaining six cases, in which one island has experienced a shock to money supply of a different sign to the shocks hitting the other two islands. As a result, the island that gets out-voted must bear the burden of monetary policy being conducted in such a way as to amplify the shock the out-voted region has been hit by, rather than countering it.

However, the two voting regimes here analyzed differ in another important regard. First of all, notice that when a party gets out-voted in the Transparent Voting regime, its money supply deviates from target by an amount equal to $2\overline{\epsilon}$ in either direction. On the other hand, under Secret Voting the maximum deviation of money supply from its target in each island is equal to $\frac{4}{3}\overline{\epsilon}$ in either direction since monetary policy must also partially reflect the preferences of the out-voted country in this case.

Though the expected deviation on each island of money supply from the target in each island is the same across the two regimes, Transparent Voting implies that the money supply in each island has a greater variance around its expectation $\overline{M}$ than under Secret Voting, as it can be verified comparing equations (2.3.33) and (2.3.37).

What is the implication of Tables 2.3 and 2.4 for the volatility of aggregate demand in each regime? By ploughing (2.3.25) into (2.3.23) we derive the following expression for aggregate demand (defining $exp[f(x)]$ to stand for $e^{f(x)}$):

$$C_m^* = \frac{d}{1-d}\frac{M_m^0}{exp\left[\frac{log(\overline{w})+(\alpha-1)logM_m^0+(\alpha-1)log\left(\frac{c}{1-c}\right)+log(\alpha)+log\left(\frac{\sigma}{\sigma-1}\right)}{\alpha}\right]}; \tag{2.3.38}$$

We have previously observed that the assumption of wage stickiness implies that money is non-neutral. In fact, as a result of wage stickiness an increase in the money supply does not feed into a one to one manner into a rise in the price level. Therefore, aggregate demand in each island is rising in the money supply at the island-wide level. As Transparency Voting is associated with a greater variance of money supply at the island-wide level, the following remark follows:

Remark 2.3.1. ***(The Volatility of Aggregate Demand and the Voting Rule:)*** *The volatility of aggregate demand at the island-wide level is higher under Transparency Voting Rule than under Secret Voting.*

The reason for the which aggregate demand is more volatile under Transparency Voting is worth re-iterating. The stickiness of wages implies that a rise (fall) in the money supply acts to increase (decrease) aggregate demand. However, monetary policy is not very effective in countering such fluctuations in aggregate demand whenever a country gets out-voted.

Secret Voting partially insures each island against the risk of being out-voted: when an island experiences an idiosyncratic supply shock of different sign to the one experienced by the other islands, its preferences still carry some weight under Secret Voting. Not so under Transparent Voting in which the median voter prevails without paying any attention to the preferences of the out-voted parties.

Therefore, the higher variance of money supply in each island under Transparency Voting implies that labor supply and consumption are also more volatile under Transparent Voting than under Voting Secrecy.

This higher amount of volatility in macroeconomic fundamentals under Transparent Voting, we show next, gives agents a greater incentive not to geographically specialize production when individual voting records are published under the Transparent Voting Regime, but rather to locate widely across all islands of the Monetary Union.

2.3.3 The Location of Industry and the Choice of Monetary Policy Regime

We aim in this section to study the link between the choice of the Voting Rule for Monetary Policy and the incentive for each household to pay a cost of magnitude τ and locate its productive and consumption activities in all the three regions of the Monetary Union, rather than geographically specializing production and consumption in a single region.

We have observed in Remark 2.3.1 that Transparent Voting involves a higher variance of aggregate demand in each island. Does the fact that the volatility of aggregate demand is higher under Transparent Voting imply that there is a greater incentive under Transparent Voting than under Secret Voting for agents to locate their economic activities widely in all the three regions of the Monetary Union? We answer this question in the affirmative after studying the issue in the following proposition.

Proposition 2.3.1. ***(Transparent Voting Induces Geographic Hedging):*** *There always exists a symmetric equilibrium in which all agents either locate consumption and production widely across all regions of the Monetary Union or in which all agents carry all of their economic activities in the island where their productive activities have a comparative advantage and therefore each industry locates narrowly in a single region.*

The threshold value for τ that induces households to locate production widely is higher under Secret Voting than under Transparent Voting, so that Transparent Voting makes widespread industrial location across all regions in the Monetary Union more likely.

Proof. First of all, we want to determine what is the actual value of the maximized utility function for an agent locating only in island m, which we denoted as $U_{1,m}$, after that the shocks to aggregate demand have taken place and monetary policy has been determined.

We do so by substituting the general equilibrium level for output of (2.3.21), for employment of (2.3.22) and for consumption of (2.3.23) into the utility function for each agent of (2.3.1) to obtain:

$$U_{1,m} = \frac{3}{r}\frac{1}{1-d}\frac{M_m^0}{P_m} - c_0\frac{1}{r}\left(\frac{3M_m^0}{P_m}\right)^{ab}; \tag{2.3.39}$$

with:

$$c_0 = \left(\frac{d}{r(1-d)b}\right)^{ab}; \tag{2.3.40}$$

Note that by locating all of her productive activities, consisting of the three goods she produces, and her consumption in only one island, the agent accounts for a share of $\frac{3}{r}$ of the economy of the island where she locates.

In an analogous fashion, we compute the resulting utility U_3 for an agent locating in all the three islands:

$$U_3 = \frac{1}{r}\left(\frac{1}{1-d}\sum_{m=1}^{m=3}\frac{M_m^0}{P_m}\right) - c_0\left(\left(\frac{\sum_{m=1}^{m=3}M_m^0}{P_m}\right)^{\alpha}\right)^{b} - \tau; \tag{2.3.41}$$

We notice that if agent i pays cost τ and locates in all the three islands, she will enjoy a share $\frac{1}{r}$ of aggregate profits, employment and money supply in each island.

Taking expectations and subtracting equation (2.3.41) from (2.3.39) we obtain:

$$E(U_3) - E(U_{1,m}) = -\tau + c_0\left[E\left(3\frac{M_m^0}{P_m}\right)^{\alpha\beta} - E\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right]; \tag{2.3.42}$$

Consider the existence of a Nash equilibrium in which all agents locate all of their productive and consumption activities evenly across the three regions of the Monetary Union. Therefore they pay cost τ and $r_m = r$ in each island. Firms have an incentive not to deviate from such equilibrium if and only if $E(U_3) - E(U_{1,m}) > 0$, that is, using (2.3.42), if and only if:

$$\tau^{tr} < c_0 \left[E\left(3\frac{M_m^0}{P_m}\right)^{\alpha\beta} - E\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right] \tag{2.3.43}$$

In fact, the left hand side of (2.3.43) captures the benefit of deviating from such Nash equilibrium, which consists of saving the cost τ necessary for being able to locate economic activities in all the three islands.

The costs of deviating from such Nash equilibrium, instead, is captured by the right hand side of (2.3.43). This captures (treating now τ as a sunk cost) as the expected difference between the utility of narrow location and that of widespread location, as derived in equation (2.3.42).

Conversely, by a similar reasoning we observe that no agent has an incentive to deviate from an equilibrium involving a single location for all agents whenever $E(U_3) - E(U_{1,m}) < 0$, which implies that locating in a single region is a Nash equilibrium whenever:

$$\tau^{tr} > c_0 \left[E\left(\frac{3M_m^0}{P_m}\right)^{\alpha\beta} - E\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right]; \tag{2.3.44}$$

Denote with $E^{tv}[x]$ and $E^{sv}[x]$ the expectations operator for variable x under Transparent Voting and Secret Voting respectively.

We aim to show that the right hand side of (2.3.43) and of (2.3.44) (that is, the cost of not diversifying geographic location) is higher under Transparency Voting than under Secret Voting, so that:

$$\left[E^{tv}\left(\frac{3M_m^0}{P_m}\right)^{\alpha\beta} - E^{tv}\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right] > \left[E^{sv}\left(\frac{3M_m^0}{P_m}\right)^{\alpha\beta} - E^{sv}\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right] \tag{2.3.45}$$

where it can be recalled that:

$$log(P_m) = exp\left[\frac{log(\overline{w}) + (\alpha-1)logM_m^0 + (\alpha-1)log\left(\frac{c}{1-c}\right) + log(\alpha) + log\left(\frac{\sigma}{\sigma-1}\right)}{\alpha}\right]; \tag{2.3.46}$$

Verification of (2.3.45) is carried out through the computations appended in appendix (A.1) confirming that indeed the above relationship holds, so that Transparent Voting requires a higher threshold value of τ than Secret Voting for agents to locate all of their consumption and productive activities in a single region. In other words, the choice of a Transparent Voting Regime over that of a Secrecy Voting one makes it more likely that agents choose to pay the cost τ and locate consumption and production across all regions of the Monetary Union. □

An intuitive account for the results of this section can now be given. Since the utility of consumption and money balances taken together are homogenous of degree one, the first terms of (2.3.39) and (2.3.41) are both linear. Therefore the welfare comparison of equation (2.3.43), whereby the agent compares the welfare gain from locating widely to the cost of doing so, rests solely upon comparison between the expected dis-utility of labor with widespread geographic location with the labor dis-utility agents have to bear by locating widely in a single area. Such comparison, in turn, rests solely upon the comparison between the volatility of aggregate demand in both scenarios.

But why is the welfare of the representative agent diminishing in the volatility of the level of aggregate demand in the island where she locates her productive activities? Since labor is subject to diminishing returns to scale, agents would prefer to be able to carry out labor smoothing. However, the greater the degree of fluctuations in aggregate demand, the more the quantity of labor supplied by each agent fluctuates across the different stochastic contingencies. Therefore, the welfare of the representative agent is diminishing in the volatility of aggregate demand.

Why do agents experience a higher expected welfare, once τ becomes a sunk cost, by locating widely? For wide location allows them to smooth out labor, since when one region experiences very high (or low) aggregate demand, there is a chance than another region may experience less macroeconomic overheating (or recessionary forces).

Conclusively, why are the benefits from locating widely higher under Transparent

Voting? This is so for under Transparent Voting fluctuations in aggregate demand in a single island are higher than under Secret Voting, implying that agents have a greater incentive to try to locate widely as to be better able to carry out labor smoothing across different stochastic regimes for the level of aggregate demand.

2.3.4 Implications for The Degree of Asymmetry of Supply Shocks

We are now ready to explore the implications of the results of Proposition 2.3.1. We aim to show that the link between monetary policy and the choice of industrial location we have just studied entails that supply shocks may grow more asymmetric inside a Monetary Union if Secret Voting is adopted, as we note in the following remark:

Remark 2.3.2. ***(The choice of Voting Regime and the Symmetry of Supply Shocks:)*** *Let us assume that all household whose goods can be produced in island m without having to pay the penalty cost τ operate in the same industry. In other words, we assume that the production of all goods in the same industry enjoys a comparative advantage in the same island. Then Proposition 2.3.1 implies that Transparent Voting has the effect of making industrial structure more uniform across different regions of the Monetary Union (as Transparent Voting may induce firms in the same industry to produce widely across all regions of the Monetary Union rather than locating all activities in the island in which a given industry enjoys some comparative advantage). This may imply that Transparent Voting could have a welfare rising effect by reducing the asymmetry of supply shocks across regions of the Monetary Union.*

In fact, we have argued in Section 2.2 Secret Voting is welfare superior to Transparent Voting *as long the degree of asymmetry of supply shocks can be held exogenous to the choice of monetary policy regime* . However, we have now derived a a framework in which Transparent Voting has the effect of making it more likely that firms locate in all regions of the Monetary Union rather than specialize production in one location. Therefore the symmetry of supply shocks would be lower under Transparent Voting than it is under Secret Voting.

Then we cannot conclude that Secret Voting is unambiguously welfare superior in the context of the problem studied in Section 2.2 once the choice of industrial location is made endogenous to the choice of Monetary Policy Regime.

In fact, we can still maintain in the context of the model developed in Section 2.2 that Secret Voting is welfare superior to Transparent Voting *if the degree of asymmetry in supply shocks is the same across the two regimes.* However, the results of Proposition 2.3.1 indicate that Transparent Voting can induce a lower degree of supply shocks asymmetry than Secret Voting by increasing the incentive for firms to locate widely across the Monetary Union. Therefore, once we study the link between the choice of voting regime and the location of industry the welfare comparison among the two regimes for voting transparency becomes, at least in theory, ambiguous.

2.4 Conclusions and Discussion

Is the assumption maintained by the ECB that Transparent Voting induces partisan monetary policy voting behavior sufficient to conclude that Secret Voting is welfare rising in a Monetary Union ?

We find in Section 2.2 that such question can be answered in the affirmative if we hold the decision of industrial location not to be affected by the rules according to which monetary policy is conducted. In fact, at this first level of the analysis, the ECB's statement almost seems to be tautologically true since it implies that under Secret Voting, unlike under Voting Transparency, monetary policy is conducted by a benevolent social planner.

However, the welfare optimality of Secret Voting becomes ambiguous, we show in Section 2.3, if we let firms' decision on where to locate be affected by monetary policy. In fact, Transparent Voting makes aggregate demand more volatile in each region, which may induce agents to locate production widely as to hedge the macroeconomic volatility induced by Transparent Voting.

The economic geography conclusion that firms in the same industry are more likely to locate production widely under Voting Transparency, rather than producing all from the same location, has the macroeconomic consequence that the asymmetry of output supply shocks in a Monetary Union may be lower under Voting Transparency than it is under Voting Secrecy.

Hence, we argue, we cannot be certain of what Voting Transparency Regime is optimal for the achievement of the Central Bank's goal even if we take the ECB's statement at face value.

Note that our findings cannot be directly compared to the arguments put forward by Krugman (Krugman 1991) suggesting that a Monetary Union may induce output supply shocks to become more *asymmetric* rather than *symmetric across countries.* The research question proposed by Krugman compares the symmetry of output supply shocks in a Monetary Union to the outcome obtaining under independent monetary policy. We, instead, compare the symmetry of output supply shocks obtaining under Voting Transparency in a Monetary Union as opposed to the asymmetry of output supply shocks under Voting Secrecy.

It must also be noticed that our findings would not be robust to the possibility that firms might hedge completely macroeconomic risk by purchasing a set of financial instruments. In fact, if this is the case, then firms would have no incentive to hedge against output fluctuations in a given country by locating widely. However, even if financial markets were complete, such hedging (especially if all firms were to try to implement it at the same time) might be costly and firms might find that to locate industrial production widely is a cheaper way of hedging macroeconomic risk than buying financial instruments.

Note also that there is a very compelling reason to explain why in practice the hedging of aggregate demand volatility is not feasible and represents a missing market. In fact, it is very difficult for firms to apply standard option pricing techniques to the hedging of aggregate demand volatility since there does not exist a traded asset with which to hedge one's position in aggregate demand. As a result, the issuer of an aggregate demand volatility derivative (the insurer) would not be able to re-insure against its positions locking in the option's premium.

Furthermore, we have omitted to consider the stabilizing effect of spillovers. As countries trade with each other, a proportion of the asymmetry in macroeconomic cycles would be self-correcting as demand in the countries growing above trend should also stimulate export demand for countries growing below trend. However, this assumption is without loss of generality as long as spillover effects are not strong enough to remove the asymmetry of fluctuations among member countries.

Our findings have also abstracted from a number of factors that are deemed in the literature to play an important role in the choice of Voting Transparency Regime. In fact, we have abstracted from the fact that some members of the Policy Committee may be more prone to suffer from the time-consistency problem, as assumed by Sibert (Sibert 1999), who shows that under such assumption Voting Secrecy may be welfare optimal.

We also abstracted from the assumption formulated by Gersbach and Hanh (Gersbach and Hahn 2000) that members of the Policy Committee may have different ability. In this context, Voting Transparency could be a device to ensure that the most efficient members are re-appointed (though one might observe that inefficient members can just emulate efficient ones under Voting Secrecy, a mechanism that may lead to the same effectiveness under Voting Secrecy as under Voting Transparency in ensuring that only efficient members set monetary policy).

We have also not analyzed an important remark by Buiter (Buiter 1999) according to which Voting Secrecy may substantially increase the power held by President of the Committee. Under this light, Voting Secrecy might turn the policy process from a Collegiate framework (the style that seems to characterize the Bank's of England Monetary Policy Committee) to a Presidential one (the style that seems to apply at the FED).

These factors seem important, and we have abstracted from them only because our objective lies in analyzing the ECB's statement in a framework that would be specific to a Monetary Union populated by agents not immune to partisans pressures. If partisan pressures lead to a conduct of monetary policy producing excessive volatility in macroeconomic fundamentals, this chapter argues, there exists, at least in our narrow and purely qualitative framework, an effect by which such volatility might be self-correcting.

Chapter 3

Interest Rates as a Vehicle of Information: The Information Signaling Problem of Monetary Policy when Central Banks Must Prevent Panic or Exuberance

Abstract

We investigate in this chapter the effects of information secrecy in a setting in which the Central Bank is endowed with asymmetric and superior information as to the path of macroeconomic fundamentals. Agents assess their disposable income and form consumption plans by using monetary policy as a signal of the Central Bank's private information. We show that in this setting counter-cyclical monetary policy risks triggering off some pro-cyclical wealth effects.

We show that gradualism or inertia in the setting of interest rates can be optimal for they allow the Central Bank to stabilize the consumption and investment behavior of agents when a pooling equilibrium applies to the signaling game. We also find that limit pricing can be optimal so that interest rate movements under asymmetric information can be smaller than under information transparency even when the Central Bank reveals its private information to agents through a separating equilibrium. We interpret this result by analogy with Milgrom's and Robert's limit pricing concept (Milgrom and Roberts 1982).

We show that the choice of information transparency over information secrecy and the mandate that the Central Bank should publish detailed minutes of its meetings render interest rates more volatile and imply that interest rates are in each period less likely to stay on hold. We show that information secrecy can be welfare optimal in our model when capital income expectations receive a relatively large weight in the determination of consumption plans. We also derive conditions under which information secrecy is welfare diminishing. We formulate a conjecture that our model is consistent with a high continuations to total changes ratio which we illustrate with an example.

KEYWORDS: SIGNALING EFFECT OF MONETARY POLICY, INFORMATION TRANSPARENCY, ASYMMETRIC INFORMATION IN MONETARY POLICY.

3.1 Introduction

Consider the following scenario: The Central Bank, which holds asymmetric information on the future path of macroeconomic fundamentals, forecasts a negative output shock in the near horizon. Agents form their consumption and investment plans conditioning upon their expected disposable income. The Central Bank is tempted to lower interest rates with the view of boosting investment. And yet, rates remain on hold.

Were interest rates to move, agents would understand that a negative shock has hit their financial portfolio; consumption would then respond to an interest rate cut in a way that only amplifies the shock that the Central Bank was trying to counter-act by lowering rates. In order not to signal to agents the shock it has detected, the Central Bank decides not to lower rates immediately in spite of the forthcoming recession.

This scenario provides the starting intuition for the analysis of this chapter which investigates the problem of information transparency in the setting of a signaling model for monetary policy. Information transparency is interpreted as capturing the degree upon which a Central Bank shares with agents its assessment of the outlook for macroeconomic fundamentals. This consists of both a wealth of information and an interpretation of the available evidence which translates data into a qualitative or quantitative assessment for the macroeconomic outlook.

It can be recalled at this stage that the FED divulges its macroeconomic forecasts with a lag of five years. Such forecasts, presented at each FOMC meeting usually in the form of a median value, summarize predictions for output and inflation by members of the FED's staff, the FED's structural model and the members of the FOMC. An unsuccessful lawsuit was placed against the FED in the 80's to force it to divulge immediately its macroeconomic forecasts (an account of which is given by Goodfriend (Goodfriend 1986)). The FED successful opposed the lawsuit by arguing that information transparency would have caused harmful volatility in financial markets.

Information transparency has giving rise to a recently burgeoning literature. However, the investigation of information transparency would be a surreal exercise if Central Banks were not endowed with any superior information on the path of macroeconomic fundamentals. Therefore, before proceeding to any further consideration, we present and assess the available evidence on the fact that Central Banks are endowed with asymmetric and superior information as to the path of macroeconomic fundamentals.

Recent research by Christina and David Romer (Romer and Romer 1996) and (Romer and Romer 2000) investigates empirically both the existence of private information for the FED and its source. It is concluded that: i) the FED is endowed with private information on the future outlook for inflation and output; ii) and that such informational advantage for the Central Bank does not stem from the fact the Central Bank enjoys superior information as to the likely path of monetary policy.

The first conclusion is reached by regressing private sector's forecast errors on both inflation and GDP on their discrepancy with respect to FED's forecast errors (which are kept secret for five years). It is found that the whole fitted forecast error by the private sector equals, on average, the amount by which private forecasters departed from the FED's predictions.

Was the source of the informational advantage stemming for the fact that the FED is only endowed with a sheer asymmetric knowledge about its own policy, rather than on the path of macroeconomic fundamentals, than we would observe that: (a) private sector's over-predicts output and inflation whenever the FED tightens by surprise; (b) on the converse, the private sectors predictions as to output and inflation would be lower than the FED's forecasts whenever an unanticipated monetary ease takes place.

The data, Romer and Romer argue, display exactly the reverse pattern: when the FED tightens by surprise, its forecast of inflation lies above private agents' ones; when it instead lowers by surprise base rates, conversely, its forecasts of inflation are lower than the projections of the private sector. The authors deem their findings conclusive of the fact that FED's actions should signal important macroeconomic information to agents, precisely because FED's behavior does not reflect superior information solely on its own policy actions, but rather on the path of macroeconomic fundamentals.

Though this study is very encouraging in ensuring that the literature on information transparency is motivated, we would like to put forward some qualifications. First of all, only one empirical study has been so far carried out and hence the empirical testing of the existence of asymmetric information between agents and Central Banks still lacks a wide and diverse base of investigation. Secondly, the mentioned study only focuses on the US economy and that its implications extend to other OECD countries can be conjectured but not substantiated. However, such initial finding seems to be consistent with the casual observation that no other agent devotes the same amount of resources to macroeconomic forecasting as Central Banks and as a result Central Banks' information on the evolution of macroeconomic variables is likely to be superior.

Having justified the assumption that Central Banks are endowed with asymmetric information, we now proceed to discuss the main results of the information transparency literature whose research agenda rests on the crucial assumption that Central Banks are endowed with asymmetric information as to the path of macroeconomic fundamentals. The main focus of the analysis lies in the welfare comparison between information secrecy and trasparency, however the welfare results vary according to the specific framework studied in each specific research exercise. We would like to organize the literature into three sub-families: i) models based on a Lucas surprise function and a time-consistency inflation bias ((Faust and Svensson 2000),(Geerats 2000)); ii) models assuming a Lucas style supply function but characterized by a time-consistency inflation bias (first model of Cukierman (Cukierman 1999),(Gersbach 1998)); iii) Keynesian frameworks in which output is demand determined (such as the second model in Cukierman (Cukierman 1999) and the work of Jensen (Jensen 1999)).

Note that a number of the papers above also assume the agents are imperfectly informed about the loss function of the Central Bank ((Faust and Svensson 2000),(Geerats 2000),(Jensen 1999)).

A brief account of the literature could summarize the pattern of the results of each sub-family of models as follows: i) information secrecy is welfare diminishing when the time-consistency bias is considered. In fact, in this case information secrecy makes agents' inflationary expectations less sensitive to the Central Bank's actions that under information transparency, which worsens the inflationary bias of monetary policy (note that in Geraats' model (Geerats 2000) the Central Bank, while boosting output above its natural level, does not attempt to stabilize it) ; ii) in a Lucas supply function framework not characterized by a time consistency bias information secrecy is welfare superior for it diminishes the volatility of agents' inflationary expectations and it then allows the Central Bank to stabilize output; iii) in Jensen's model (Jensen 1999) information transparency has the effect of forcing the Central Bank during her first periods in office to place a higher weighting on inflation stabilization than it would otherwise do. This is so for the Central Banker needs to signal to agents that it is highly committed to controlling inflation. The welfare comparison between information secrecy and transparency in Jensen's model is ambiguous. Instead, in the second model of Cukierman (Cukierman 1999) welfare secrecy is always welfare rising for transparency makes agents inflationary expectations more volatile, which rises the volatility of real interest rates, even though information transparency does not alter the volatility of output and inflation.

Our analysis differentiates itself from the existing literature in two important regards. First of all, we do not assume that the only area of interaction between the monetary policy strategy chosen by the Central Bank and the private sector lies in the private sector's inflationary expectations. As observed by the FED's vice-chairman and distinguished economist Alan Blinder ((Blinder 1997),p.8), this setting seems overly restrictive. Instead, we study a notion somewhat reminiscent of *the animal spirits of the investors* concept first described by Keynes. We posit that in our framework of asymmetric information but full rationality agents' assessment of their disposable income depends on the signals learnt from the conduct of monetary policy. Hence, rational agents let their consumers' confidence depend upon the observed monetary policy stance, which then affects the incentives of the Central Banker.

Secondly, we aim to relate to our framework a broad set of questions, including gradualism, inertia, the reversals to total changes ratio and limit pricing behavior.

The rest of the chapter is organized as follows. We develop the signaling monetary policy game framework in Section 3.2. We define the adopted solution concept and provide a simple equilibrium example in Section 3.3. We then draw the macroeconomic implications for our model and report some very simple simulations results (which have a qualitative but not quantitative interpretation) in Section 3.4. We conclude and discuss our results in Section 3.5.

3.2 The Framework of the Model

The monetary policy game we model has the following sequential structure: a) Nature determines an output shock denoted as ϵ_t; b) the Central Bank, endowed with perfect knowledge as to the magnitude of the output shock ϵ_t, sets the real rate r_t; c) Unlike the Central Bank, agents are incompletely informed and hence ignore the magnitude of the output shock ϵ_t; however, agents employ the monetary policy *signal* sent by the Central Bank to form expectations as to the actual magnitude of ϵ_t. Agents, hence, condition their consumption decisions on the expected magnitude of the output shock since this is expected to feed upon their wealth. A refinement criterion will be introduced to impose some structure upon agents' beliefs.

The high-level structure of the game is depicted in Figure 3.1. We have turned the game of *incomplete information* (where the receiver ignores her type) into one of *imperfect information* (where the receiver ignores her exact position in the game tree). This transformation, due to Harsanyi (Harsanyi 1968), is an often employed expedient which does not bring about any loss of generality (see, for instance, Fudenberg and Tirole (Fudenberg and Tirole 1991), p 209).

This section analyzes each step of this sequence in order to write out a payoff to the game for the Central Bank which is a function of the following three variables: the type for the output shock (ϵ_t); agent's expectation of the output shock once monetary policy is observed denoted as $E\left[\epsilon_t \middle| \Delta r_t\right]$; and finally the message ϵ_j the Central Bank sends to agents when it sets rates. Writing out the payoff for the game in such way paves the way for the numerical solution to the model we carry out.

In the spirit of a backwards induction solution, we start from the last move in the game. We first derive agents' reaction function to monetary policy in Section 3.2.1. This allows the Central Bank to anticipate what is the level of consumption and investment agents set given a certain level of interest rates and a certain level for consumers' confidence. In turn, the Central Bank uses such information to determine by backwards induction what is the level of aggregate demand stemming from any given monetary policy decision.

We then shift the focus of the analysis in an upward direction in the tree of the extensive game representation of Figure 3.1. We, in fact, then specify in Section 3.2.2 the objectives and the constraints faced by monetary policy. We then let the Central Bank perform backwards induction employing the results of Section 3.2.1 on agents' reaction

ϵ_t denotes the random
shock to agents's cash flows

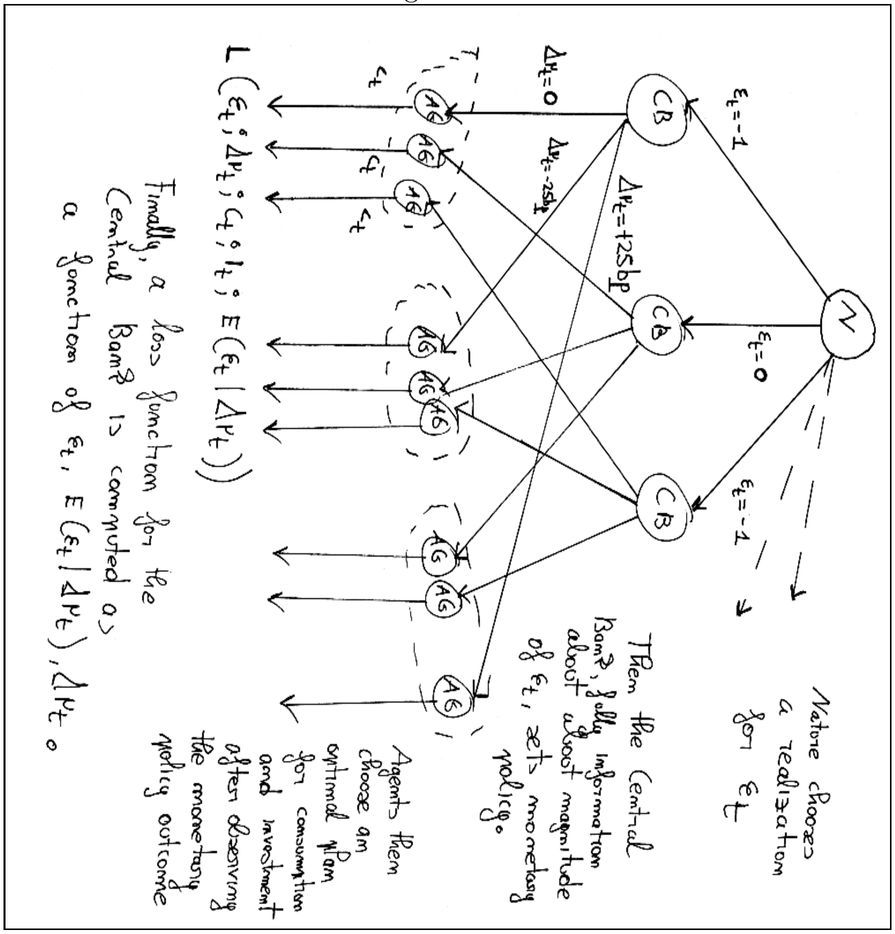

Figure 3.1: The High Level Structure of the Signaling Game

function so that we can finally derive the payoff of the game in section 3.2.3 which links the type of the output shock, agents's beliefs on the output shock and the monetary action by the Central Bank (its message) to the Central Bank's final loss function.

3.2.1 Monetary Policy and Consumers' Confidence: Agents' Reaction Function to the Interest Rate Announcement

We aim in this section to study the link between innovations to monetary policy, consumers' confidence as captured by their expectations on life-time disposable income and consumption. Our final objective lies in deriving a reaction function to describe how the level of consumption responds to the monetary policy signal agents receive from the Central Bank.

We tackle this task in two steps. We first model in Section 3.2.1.1 how agents determine their disposable income given a specific belief on the magnitude of all shocks to firms' cash flows. We then investigate in Section 3.2.1.2 the process by which agents employ their expectations as to the level of disposable income to determine aggregate consumption.

It might be useful at this stage to illustrate at an informal level the intuition driving the results of this section. Consider the following mechanism that translates a change in interest rates to a revision in agents inter-temporal optimal consumption plans via wealth effects.

The Central Bank announces a change in rates *(Δr_t)*. Agents *optimize their consumption plans by extrapolating information* as to their future wealth from Central Bank's behavior. Central to the mechanism lies the assumption that the Central Bank has perfect knowledge of all the output shocks hitting the economy. Agents exploit such information as to try to smooth their consumption path appropriately.

Agents, in fact, revise their consumption in the upwards direction if they think that the Central Bank has, through its decision, signaled that a positive temporary innovation to their disposable income (denoted by ϵ_t) is likely to take place. Conversely, consumption plans are curtailed following an announcement about monetary policy that makes agents revise downwards their expected wealth.

The final aim of this section is to derive an aggregate consumption function of the form:

$$\mathbf{c_t} = \frac{1}{2}\left\{\mu(\rho)\epsilon_t + \sigma\left[\mu(\rho)E\left(\epsilon_t \middle| \Delta r, t\right)\right]^{a2} + \hat{c}\right\};\ 0 < \sigma \leq 1;\ 0 < \alpha 2 \leq 1; \qquad (3.2.1)$$

The notation must be interpreted as follows. The output shock which feeds on agents's cash flows is denoted with ϵ_t, while changes in the real interest rate are captured by Δr_t; the term $\mu(\rho)$ is increasing in the persistence of the temporary shocks to output; all other terms are subsumed in the constant term $\hat{c}$ while the interpretation of the other parameters is illustrated as we proceed with the derivation of (3.2.1).

3.2.1.1 How Agents Determine Expected Disposable Income

We initially detail the mechanism that allows for interest rates announcements to have wealth effects and to feed upon consumption plans, and then incorporate expected disposable income in a simple dynamic programming problem to derive Euler equations *á lá* Hall (Hall 1978) and determine a solved out consumption function.

The economy is composed by n identical firms and n agents. Each agent i is employed by one firm in sector j. Let *cash flow* $R_{j,t}$ for the firm j in period t be equal to a time-invariant term $\overline{R}$ plus an autoregressive innovation innovation $\epsilon_{j,t}$ which depends on a output shock whose aggregate magnitude before period t is only known by the Central Bank.

$$R_{j,t} = \overline{R} + \epsilon_{j,t} \quad \forall j; \tag{3.2.2}$$

$$\begin{aligned} \epsilon_{j,t} &= \rho\ \epsilon_{j,t-1} + v_{j,t}; \rho < 1; \\ v_{j,t} &\sim IN(0, VAR_v); \\ \epsilon_{j,t-1} &= 0 \quad \forall j; \end{aligned} \tag{3.2.3}$$

The assumptions jointly imply that shocks impacting $R_{j,t}$ die out slowly. In the limit case in which $\rho = 1$, cash flows follow a unit root martingale process so that:

$$E[R_{j,t+s}] = R_{j,t}\ \forall s \geq 0;$$

Capital holders and workers engage into symmetric Nash bargaining game over profits. Therefore, one-half of each firm's profits go to the single worker each firm employs and one half to the share-holders. All share-holders split their portion of the profits symmetrically among themselves.

Agents possess incomplete information over the real shocks that hit output and cash flows in a way that we now formalize:

Assumption 3.2.1. ***(Asymmetry of Information between Central Bank's and Agents)*** *Agents have imperfect information over the magnitude of ϵ_t. Specifically, we assume that each worker j has complete knowledge about $\epsilon_{s,t}$ for $s = j$, the shock that has hit the cash flow of the firm by which she is employed. However each agent j does* *not know $\epsilon_{s,t}$ for any $s \neq j$ and therefore has no information about the shocks that have occurred to the firms in other sectors. As a result, agents enjoy perfect knowledge about their labor income, while they must condition their capital income expectations upon the signals that the Central Bank sends through monetary policy.*

On the other hand, we assume the Central Bank to know the magnitude of the output shock ϵ_t.

Each agent owns a stake $\frac{1}{n}$ of each firm. The representative consumer retains a share σ of the shares in the domestic economy, and trades the rest for foreign assets. This entitles her to a share $\sigma\frac{1}{2n}$ of the cash flow of each firm under a symmetric bargaining game with the only employed worker in each sector.

Profits are taxed in a progressive fashion. Hence expected disposable capital income will be equal to $E[(stochastic\ cap\ income)^{a2}]$ with $a2 < 1$; a2 is falling in the degree of fiscal progressiveness. We assume, for simplicity, that the time-invariant portion of cash flows $\overline{R}$ is not taxed. Note also that a2 is a ratio with an odd number both at the numerator and at the denominator so that disposable income is always defined.

The scenario depicted implies that the worker i first assesses her own disposable income at period t by looking at the shock that she has observed in her own sector j, and then forms expectations as to the magnitude of the aggregate shock after that the monetary stance Δr_t is known. Equation (3.2.4) describes the accounting formula by which income expectations are computed given any belief on the shocks to output.

We denote with $y_{i,t}$ the disposable income for agent i at time t, which consists of three components, described in the order by which they appear in (3.2.4); the first component captures the stochastic component of labor income; the second reflects the stochastic component of capital gains net of taxation; the third term mirrors income from foreign assets, which we trivialize to being non-stochastic and time-invariant, together with the time-invariant and tax-free component of firms' cash flows $\overline{R}$. We can therefore describe disposable income in the following manner for the i worker employed by firm j:

$$E\Big[y_{i,t}\Big|\Delta r_t\Big] = \frac{1}{2}R_{j=i,t} + \frac{\sigma}{2n}E\Big[\Big(\sum_{j=1}^{n} R_{j,t}\Big|\Delta r_t\Big)^{a2}\Big] + \hat{c}_0; \quad \forall i;$$
$$= \frac{1}{2}\epsilon_{j,t} + \frac{\sigma}{2n}E\Big[\Big(\sum_{j=1}^{n} \epsilon_{j,t}\Big|\Delta r_t\Big)^{a2}\Big] + \hat{c}_0 \quad ; \ \sigma \leq 1; \tag{3.2.4}$$

We can also assume that capital income is partially insurable. Agents pay an insurance fee of magnitude F which allows them to: i) receive the expected level of their capital income with the expectation being updated upon the latest monetary policy observation; ii) to hedge their portfolio returns from the change in the yield of liquid savings stemming from a change in real interest rates. However, agents cannot insure their labor income before the monetary policy stance allows insurers to refine their expectations on capital income. Hence under this assumptions (3.2.4) becomes equivalent to:

$$E\Big[y_{i,t}\Big|\Delta r_t\Big] = \frac{1}{2}\epsilon_{j,t} + \frac{\sigma}{2n}\Big[E\Big(\sum_{j=1}^{n} \epsilon_{j,t}\Big|\Delta r_t\Big)\Big]^{a2} + \hat{c}_0 - F \quad ; \sigma \leq 1; \tag{3.2.5}$$

While labor income is known with certainty to the worker employed by the firm j to be equal to the idiosyncratic shock in the sector j, the link between Central Bank's actions and income expectations hinges crucially on wealth effects which are unknown to agents, who can only form expectations on wealth effects via conditioning upon monetary policy.

When simulating the model, we employ $a2 = 0.8$ as a benchmark in most of the scenarios investigated by the thesis, so that we hold the taxation regime to be nearly linear. In fact, the results hold without loss of generality even for a linear capital income regime.

Having determined the link between the information extracted through monetary policy and agents' expectations on their capital income, we turn attention to derive a solved out consumption function.

3.2.1.2 Consumption and Monetary Policy

We now incorporate the permanent income expectations derived in equation (3.2.5) in an inter-temporal utility maximization model of consumption, which follows Hall (Hall

1978), so that we can derive a solved out consumption reaction function. This reaction function informs allows the Central Bank to anticipate what level of aggregate demand shall result from each possible monetary policy decision.

The consumer i is endowed with a quadratic utility function, which she optimizes for a planning horizon of T periods subject to a discount rate δ of time invariant magnitude so that the consumer seeks to maximize:

$$\max\ E\Big[\sum_{t=0}^{T-1} \delta^t \left(ac_{i,t} - bc_{i,t}\right)^2 \left|t\right.\Big]; \tag{3.2.6}$$

The assumption of *quadratic utility* is crucial to obtain a tractable closed form solution to the problem. In fact, this functional form allows us to treat the marginal utility of the expected level of consumption as being equivalent to the marginal utility of the certainty equivalent. This is due to the fact that the marginal utility of consumption is linear under this specification.

The stock of wealth at time t for agent i (denoted with $A_{i,t}$) evolves according to the following inter-temporal budget constraint:

$$A_{i,t+1} = (1 + r^p)\left(A_{i,t} + Y_{i,t} - C_{i,t}\right); \tag{3.2.7}$$

Other items of notation are defined as follows: $Y_{i,t}$ represents the total income accruing to the representative agent i at time t and r^p represents the rate of return to the stock of liquid savings held by the agent. This return r^p would normally be a function of the short-run interest rate. But, following a previously stated assumption, we assume for simplicity and without loss of generality that agents fully hedge the volatility in r^p imparted by the short-term rate r as part of the insurance policy they purchase at a cost of F. Therefore, r^p can be assumed not to be a function of r. Alternatively, r^p represents the yield of a long-term bond which we assume to be pretty insensitive to changes in the short-run rate.

Usual resolution techniques of dynamic programming turn this multi-period problem into a two-stages one by introducing a value function $V(A_t)$, which yields the maximum expected utility to be gained by starting the problem at time t with an initial endowment of wealth level $A_{i,t}$:

$$V(A_t) = \max \left\{ U(c_t) + \delta E\Big(V_{t+1}(A_{t+1})\Big| t\Big)\right\}; \tag{3.2.8}$$

As equation (3.2.7) implies, one additional unit of consumption in period t reduces future wealth by $(1 + r^p)$. Therefore, differentiating the right hand side of equation (3.2.8) with respect to $c_{i,t}$ we can derive the optimal value for the marginal utility of consumption in the initial period t:

$$U'\left(c_{i,t}\right) = \delta E\Big[V'_{i,t+1}(A_{i,t+1})(1 + r^p)\Big| t\Big]; \tag{3.2.9}$$

Differentiating now both sides of equation (3.2.8) with respect to $A_{i,t}$ and exploiting (3.2.9) the traditional envelope relationship is derived:

$$\begin{aligned} V'(A_{i,t}) &= \delta E\Big[(1 + r^p)V'(A_{i,t+1})\Big| t\Big]; \\ &= U'(c_{i,t}) \end{aligned} \tag{3.2.10}$$

This result implies that to measure the marginal value of an additional unit of initial wealth it is *sufficient* to compute the marginal utility of current consumption.

At this stage the assumptions that the discount rate equals to the inverse of the rate of return is usually imposed so that:

$$(\delta)^{-1} = (1 + r^p) \tag{3.2.11}$$

Exploiting this assumption and substituting recursively equation (3.2.10) into (3.2.9) the following result obtains:

$$U'(c_{i,t}) = E\Big[U'(c_{i,t+s})\Big] \quad \forall s; \tag{3.2.12}$$

Equation (3.2.12) implies that consumers equalize the expected marginal rate of utility from consumption in all future periods as a result of diminishing returns to consumption.

The assumption of quadratic utility allows us to replace in equation (3.2.12) the marginal utility of the expected level of consumption with the marginal utility of the certainty equivalence in virtue of the fact that a quadratic utility function implies that marginal utility is linear. It is implied under such framework that, along the optimal consumption path, the consumer plans ex-ante to carry out perfect consumption smoothing:

$$c_{i,t} = E\Big[c_{i,t+s}\Big] \quad \forall s; \tag{3.2.13}$$

This results states that the consumer plans to equalize consumption across all states of the world because consumption yields diminishing marginal returns.

We are now ready to derive the *optimal reaction function* for individual agents' consumption that determines how consumption responds to the aggregate output shock ϵ_t and agents' expectations of such shock $E\Big[\epsilon_t\Big|\Delta r_t\Big]$ conditional upon the behavior of monetary policy.

Remark 3.2.1. ***(The Impact of the Information conveyed by Monetary Policy on Agent's Consumption Reaction Function):*** *Agents' optimal aggregate consumption depends both upon the magnitude of the shock hitting aggregate firms' cash flows ϵ_t and on the expectation of such output shock $E\Big[\epsilon_t\Big|\Delta r_t\Big]$ computed after that agents observe the behavior of monetary policy. The aggregate consumption reaction function c_t^* takes the form:*

$$c_t^*\Big(\epsilon_t, E\Big[\epsilon_t\Big|\Delta r_t\Big]\Big) = \frac{\hat{y}}{2} + \frac{1}{2}\Big[\mu(\rho)\epsilon_t + \sigma\Big[\mu(\rho)E\Big(\epsilon_t\Big|\Delta r_t\Big)\Big]^{\alpha 2}\Big]; \tag{3.2.14}$$

where: $\mu(\rho) = \frac{r^p}{1+r^p-\rho}$; $\frac{\hat{y}}{2} = \hat{c}_0 - F$;

The aggregate consumption function of (3.2.14) acts as the reaction function of the receiver to the signal of the sender in the signaling game we model whose high-level structured is sketched by Figure 3.1.

Proof. All wealth must be exhausted by period T, when no more consumption takes place for at that stage wealth has no use. We rule out bequests. Therefore the ex-post budget constraint (which must always hold) yields the accounting identity:

$$A_{i,T} = A_{i,0}(1+r^p)^T + \sum_{s=0}^{T-1}\Big(Y_{i,s} - c_{i,s}\Big)\Big(1+r^p\Big)^{t-s} = 0; \tag{3.2.15}$$

We now nullify the effect of initial wealth by letting $A_{i,0} = 0$, which follows from having ruled out bequests. Furthermore, to obtain a tractable close form solution, we let T grow infinitely large.

Taking expectations from both sides of equation (3.2.15) and letting for analytical simplicity T grow infinitely large yields:

$$E\left[\sum_{s=t}^{\infty}\left[Y_{i,s}(1+r^p)^{t-s}\right]\right]=E\left[\sum_{s=t}^{\infty}\left[c_{i,s}(1+r^p)^{s-t}\right]\right]; \tag{3.2.16}$$

We can now factor out consumption in the left-hand side exploiting the perfect consumption smoothing result of (3.2.13) which, after using the properties of geometric series, yields:

$$c_{i,t}=\frac{r^p}{1+r^p}\left[\sum_{s=t}^{\infty}(1+r^p)^{t-s}E_{i,s}Y_{i,s}\right]; \tag{3.2.17}$$

We are now able to link the inter-temporal optimization result of (3.2.17) with the expectation of *disposable income* conditional on monetary policy derived in (3.2.5). Substituting for (3.2.5) into (3.2.17) we obtain the level for consumption chosen by each agent:

$$c^*_{i,t}\Big(\epsilon_{i,t},E\left[\epsilon_t|\Delta r_t\right]\Big)=\frac{1}{2}\left[\mu(\rho)\epsilon_{i,t}+\frac{\sigma}{n}\Big[\mu(\rho)E\Big(\epsilon_t|\Delta r_t\Big)\Big]^{\alpha 2}\right]+\frac{\hat{y}}{2n}; \tag{3.2.18}$$

And finally aggregating upon the n agents aggregate consumption turns out, as we set out to prove, to be equal to:

$$c^*_{i,t}=\Big(\epsilon_{i,t},E\left[\epsilon_t|\Delta r_t\right]\Big)=\frac{\hat{y}}{2}+\frac{1}{2}\Big\{\mu(\rho)\epsilon_t+\sigma\Big[E\Big(\epsilon_t|\Delta r_t\Big)\Big]^{\alpha 2}\Big\};\mu(\rho)=\frac{r^p}{1+r^p-\rho}; \tag{3.2.19}$$

If $\rho\approx 1$ the following useful approximation to (3.2.19) holds:

$$c^*_{i,t}\big(\epsilon_{i,t},E\left[\epsilon_t|\Delta r_t\right]\big)=\frac{\hat{y}}{2}+\frac{1}{2}\Big\{\epsilon_t+\sigma\Big[E\Big(\epsilon_t|\Delta r_t\Big)\Big]^{a2}\Big\}; \tag{3.2.20}$$

Equation 3.2.19 proves the remark and will be used as an essential building block in solving the model. □

The innovation to agents' cash flows contributes to the consumption function, which may a priori seem striking as no single individual agent knows the magnitude (and the sign) of the entire aggregate shock to cash flows. It is, in fact, assumed that private economic actors know only the magnitude of the idiosyncratic output shock that occurs to the sector in which they are employed.

As we aggregate, however, the sum of the n idiosyncratic shocks $\epsilon_{i,t}$ to each sector of the economy adds precisely up to the total shock $\frac{\epsilon_t}{2}$ to labor income. Hence aggregate labor income expectations are, through aggregation, the same that would obtain if all individual agents *pooled their knowledge on labor income* (and on labor income only) and collectively knew the economy-wide shock to labor income with the same accuracy as the Central Bank does in virtue of its asymmetric and superior information.

However, agents ignore the magnitude of the shock hitting firms' cash flows and hence they do not know the economy-wide level of the shock occurring to their capital income. As a result of this degree of incomplete information, the term $E\left(\epsilon_t \middle| \Delta r_t\right)$ enters agents optimal consumption reaction function of (3.2.19). In fact, even if agents ignore the nature of the innovations to their capital income, they still try to make inference as to the dividends they are likely to receive as to determine their permanent income expectations and hence their optimal consumption level.

After having investigated a mechanism by which interest rates carry information to agents as to their future wealth and hence affect consumer's confidence, we can state a rationale for which the Central Bank may opt not to divulge information on macroeconomic fundamentals as not to trigger off pro-cyclical wealth effects. In fact, equation (3.2.19) states that consumption spending shall be immediately reduced if agents expect monetary policy to have been eased for the Central Bank foresees a negative output shock. Hence, the Central Bank can use (3.2.19) to anticipate how its behavior could feed upon an important component of aggregate demand once the signaling game is solved.

Having established how consumption responds to monetary policy, we now turn attention to a full analysis of the framework in which monetary policy operates.

3.2.2 Objectives and Constraints for Monetary Policy

We study in this section the objectives and the constraints faced by monetary policy. The framework for the model of the economy we assume is simple and is not derived from micro-foundations. However, it aims to deliver a pragmatic framework for the analysis of policy in the spirit of an IS-LM model with which a complicated signaling model can be later simulated in Section 3.3 before implications of the analysis are drawn in Section 3.4.

We first analyze in Section 3.2.2.1 how aggregate demand is determined using the insights of Section 3.2.1.2; we then specify in Section 3.2.2.2 the simple and stylized link we assume to exist between monetary policy and inflation; and finally we state in Section 3.2.2.3 what are the objectives of monetary policy.

3.2.2.1 The Determination of Aggregate Demand

We incorporate in this section the specification for aggregate consumption derived in (3.2.19) into the determination of aggregate demand in the model.

We show that monetary policy impacts aggregate demand through two channels. The *investment channel* of monetary policy is the traditional effect whereby the investment component of the IS curve is diminishing in the cost of money. The *monetary policy signal expectation channel*, instead, captures the effect that monetary policy has on consumer confidence and hence on consumption.

Aggregate demand in our model consists of three components: government spending, investment demand and aggregate consumption. However, we trivialize government spending to take a constant value $\overline{g}$.

We let the rate of investment be directly proportional to the quantity of money the banking sector creates $I_t = \bar{i} + \phi\Delta m$, where Δm represents changes in the monetary base. We follow the results by Stiglitz and Weiss (Stiglitz and Weiss 1981) in order to assume that changes in the quantity of money affect the level of investment. It is possible to derive such results in a framework where firms intending to borrow are quantity constrained. Interest rates set by commercial banks to firms applying for loans are below money-clearing levels because of an adverse selection problem (high interest rates tend to increase the proportion of borrowers with high bankruptcy risk in the total risk-pool managed by each bank). Hence, an increase in the quantity of money allows the Banking sector to increase its lending as observed by Blanchard and Fischer ((Blanchard and Fischer 1987), p.487) since it increases the quantity of deposits held by commercial Banks at any given level of the interest rate.

The consumption component of aggregate demand is derived in equation (3.2.19) from an inter-temporal optimization problem carried out by agents facing incomplete information as to their capital income but knowing that the Central Bank carries out monetary policy being endowed with complete information.

An expression for aggregate demand obtains by summing over the three components of aggregate demand, that is the consumption component of (3.2.19), the investment component $I_t = \bar{i} + \phi\Delta m$ and finally the government component $G = \overline{g}$ and lumping constant terms into the term $\frac{1}{2}\hat{y}$ aggregate demand turns out to be equal to:

$$y_t = \frac{1}{2}\left[\hat{y} + \mu(\rho)\epsilon_t + \phi\Delta m_t + \sigma\left(\mu(\rho)E\left[\epsilon_t \middle| \Delta r_t\right]\right)^{a2}\right]; \tag{3.2.21}$$

An observation on what is the average value for aggregate demand in the model is in order anticipating some results to be later derived. It is useful to bear in mind for future reference that in equilibrium $E(y_t) = \frac{1}{2}\hat{y}$ since we also demonstrate that in equilibrium: $E(\epsilon_t) = E(\Delta r_t) = E(\Delta m_t) = 0$.

Equation (3.2.21) is to be interpreted in a fashion analogous to equation (3.2.19). First of all, notice that aggregate demand is increasing in the magnitude of the shock ϵ_t hitting firms' cash flows as labor income is also increasing in ϵ_t. Even if each individual agent, rather than having full knowledge on the magnitude of ϵ_t, is perfectly informed only about the shock that has occurred to her sector $\epsilon_{j,t}$, half of the total shock ϵ_t impacts consumers' spending before the total magnitude of the shock is revealed to agents as the aggregate consumption function aggregates over the spending plan of each agent, which incorporates the idiosyncratic shock $\epsilon_{j,t}$ occurring in the sector by which each agent is employed.

The term $\frac{1}{2}E\left[\mu(\rho)\epsilon_t \middle| \Delta r_t\right]$ contributes to the determination of aggregate demand of (3.2.21) via wealth effects, as derived in equation (3.2.19). In fact, each agent has to form expectations as to the magnitude of the shock impacting her capital income only by observing the behavior of the Central Bank, the only actor in the model enjoying full information and hence the only sender of a reliable signal as to the evolution of capital income. The only action of the Central Bank agents can observe is the setting of monetary policy, hence agents need to condition their expectation of ϵ_t upon the monetary policy innovation Δr_t.

The aggregate demand expression of (3.2.21) illustrates the two aspects of the transmission mechanism at work in the model. On the one hand, money creation affects investment. Hence, to the extent by which interest rates affect money creation, to be specified below, monetary policy feeds upon aggregate demand via the usual investment channel.

Secondly, the process of monetary policy acts as a signal for agents as to the information held by the Central Bank. Hence, the setting of interest rates affects agents' expectation as to the magnitude of their capital income, which, in turn, feeds upon consumption. If interest rates change abruptly, agents might experience panic or euphoria, which might lead to a sharp fluctuation in a component of aggregate demand.

3.2.2.2 Inflation and Money Creation

We now specify how monetary policy affects money creation and inflation.

The quantity of money held by Commercial Banks depends on the discount rate at which they can borrow from the Central Bank. We assume that the quantity of money the Banking system creates depends upon the appropriate measure of the repo rate according to the following relationship:

$$\Delta m_t = -(\Delta r_t)^{a1}; \quad a1 > 0; \tag{3.2.22}$$

Note that $a1 = \frac{z_1}{z_2}$ where both z_1 and z_2 are restricted to be odd numbers so that the expression of (3.2.22) is always defined.

We finally let changes in the price level depend upon the quantity of money created by the Banking system via a specification that nests the quantity theory of money. The parameter δ represents the speed at which changes in money feed, symmetrically in both directions, into the price level:

$$\pi_t = \delta \Delta m; \tag{3.2.23}$$

Note that the specification of (3.2.23) nests the quantity theory of money. In fact, if $\delta = 1$ and the velocity of circulation and of output is held constant, the quantity theory of money applies.

We are now ready to analyze the objectives and the constraints faced by monetary policy.

3.2.2.3 The Objectives of Monetary Policy

We not turn attention to defining the policy objective of monetary policy and the tools available to policy-makers. The Central Bank minimizes a loss function which is quadratic in the deviation of aggregate demand from a given target level and in the level of current inflation. The following loss function applies, which, for clarity, we state together with the level of aggregate demand derived by substituting (3.2.22) into (3.2.21):

$$L_t\Big(y_t, \pi_t\Big) = \Big(2y_t - k\hat{y}\Big)^2 + \beta\Big(\pi_t\Big)^2 \quad k = 1; \tag{3.2.24}$$
$$y_t = \frac{1}{2}\left[\hat{y} + \mu(\rho)\epsilon_t - \phi(\Delta r_t)^{\alpha 1} + \sigma\Big(E\big[\mu(\rho)\epsilon_t\big|\Delta r_t\big]\Big)^{a2}\right];$$

Equation (3.2.24) together with (3.2.22) and the mechanism by which agents determine $E\big[\mu(\rho)\epsilon_t\big|\Delta r_t\big]$ (which can be analyzed in the context of a signaling game) specify the problem faced by the Central Bank. The instrument of policy is Δr_t.

We set $k = 1$ throughout the analysis of this chapter to reflect the interpretation that the Central Bank tries to stabilize aggregate demand around its average level while trying to keep the price level stable.

In fact, notice that, accepting at face value at this stage our statement that in equilibrium aggregate demand is on average and in expectation equal to $\frac{1}{2}\hat{y}$ as $E(\epsilon_t) = E(\Delta r_t) = E(\Delta m_t) = 0$, then setting $k = 1$ in (3.2.24) implies that the bliss point for the Central Bank's loss function is one in which aggregate demand is equal to its target value while prices are stable.

The sequence of the actors' moves is as follows: 1) Nature chooses a type ϵ_t for the economy, which our model interprets in macroeconomic terms as the determination of a (temporary) shock to agents' cash flows of a given magnitude ϵ_t; 2) The Central Bank observes the shock to cash flows ϵ_t and hence chooses, after a complicated backwards induction process, how to set monetary policy by determining Δr_t; 3) Agents observe monetary policy and set their consumption and investment decisions. They use rational expectations to try to infer from monetary policy how to set aggregate demand according to (3.2.21). The determination of $E\big[\epsilon_t\big|\Delta r_t\big]$ using rational expectations can only take place in the context of a signaling model we analyze in section (3.3) in which the payoff of the Central Bank is described by (3.2.24) and (3.2.23).

The economy can experience two regimes: the *overheating regime* (occurring when absent active monetary policy aggregate demand would fall above its target level) and the *recession regime* (occurring when absent active monetary policy aggregate demand would fall below its target level). Setting $k = 1$ in equation (3.2.24) and considering, for illustration, the full information benchmark in which $E\left[\epsilon_t \middle| \Delta r_t\right] = \epsilon_t$, the economy is overheating from the standpoint of the Central Bank whenever $\epsilon_t > 0$, while a recessionary regime is observed when $\epsilon_t < 0$. Aggregate demand is on target without any innovation to monetary policy whenever $\epsilon_t = 0$.

The results derived in this section allow us to determine what value the loss function takes for the Central Bank for any combination of monetary policy action Δr_t and agents' expectations on capital income $E\left[\epsilon_t \middle| \Delta r_t\right]$. However, we want to transform this setting in which the Central Bank implements an action Δr_t, to one in which the Central Bank declares to an auctioneer to be of type ϵ_j, so that the auctioneer can implement the monetary policy innovation Δr_t on behalf of the Central Bank once ϵ_j is announced. We do so in the next section employing the revelation principle.

3.2.3 The Problem Faced by the Central Banker

The objective of this section lies in deriving an indirect loss function $L\Big(\epsilon_t, \epsilon_j, E\left(\epsilon_t \middle| \epsilon_j\right)\Big)$ which maps into a given value of the Central Bank's loss function any combination of: 1) a shock to cash flows of magnitude ϵ_t which is of private information to the Central Bank; 2) a message (possibly an untruthful one unless a pure separating equilibrium holds) sent on behalf of the Central Bank that a shock of magnitude ϵ_j has occurred and hence the Central Bank, for any given value of beliefs $E\left(\epsilon_t \middle| \epsilon_j\right)$ sets interest rates as it were type ϵ_j; 3) any value of $E\left(\epsilon_t \middle| \epsilon_j\right)$ agents set for their expectation of ϵ_t once they have observed the signal ϵ_j.

Note that *to derive such indirect loss function is not equivalent to solving the signaling game*. In fact, solving the signaling game implies finding an optimal signal for the Central Bank that acts on the knowledge that agents' expectations must be consistent with such signal in a manner specified by the chosen refinement equilibrium. Instead, we here fix to a given level the belief $E\left(\epsilon_t \middle| \epsilon_j\right)$ and ask, given such belief, what is the value achieved by the Central Bank's loss function of (3.2.24) for any combination of ϵ_t (the type of the Central Bank) and ϵ_j (the message sent by the Central Bank).

This procedure is an often employed strategy to turn a game of incomplete information from action space (a setting in which the Central Bank announces its choice of Δr_t) into types space (a setting in which the Central Bank reveals to an arbitrator its true type, and hence the arbitrator declares on behalf of the Central Bank -possibly untruthfully- that the Central Bank is of type ϵ_j and lets Δr_t depend upon ϵ_j). See, for instance, Fudenberg and Tirole ((Fudenberg and Tirole 1991), p. 255-256) for a discussion of the revelation principle and how this is used to turn a game from actions space to types (or messages) space.

The first step in the procedure lies in answering the following question: what is the optimal choice of Δr_t *given* that the Central Bank declares to be type ϵ_j and beliefs take a given value $E\left(\epsilon_t \middle| \epsilon_j\right)$ to be held for the moment fixed? Note the very important point that to answer this question *does not mean to identify the solution of the signaling game*, since when the signaling game is solved we must also determine what is the optimal message ϵ_j for the Central Bank to send and what is a consistent level for expectations $E\left(\epsilon_t \middle| \epsilon_j\right)$ to lie at.

To determine what is the optimal level of Δr_t given $\epsilon_j \times E\left(\epsilon_t \middle| \epsilon_j\right)$, we find the choice of Δr_t that minimizes the loss function of equation (3.2.24) for *any given* value of $\epsilon_j \times E\left(\epsilon_t \middle| \epsilon_j\right)$ and subject to equations (3.2.21), (3.2.22),(3.2.23), which yields:

$$\left(\Delta r_t^*\right)^{a1}\left(\epsilon_j \times E(\epsilon_t \middle| \epsilon_j)\right) = \frac{\phi}{(\phi^2+\psi)}\frac{1}{2}\left(\hat{y}(1-k)+\mu(\rho)\epsilon_j+\sigma\left(E\left[\mu(\rho)\epsilon_t \middle| \epsilon_j\right]\right)^{a2}\right); k=1; \tag{3.2.25}$$

$$with \quad \psi = \beta\delta^2;$$

We also state formally the central point of the discussion above that the value of Δr_t^* does not represent a solution of the signaling game, but rather the optimal choice of interest rates for any given message sent by the Central Bank and any level of beliefs by agents $E\left(\epsilon_t \middle| \epsilon_j\right)$:

Remark 3.2.2. ***(Equation (3.2.25) Does not Describe The Optimal Choice of Interest Rates):***. *It must be emphasized that (3.2.25) does not represent the solution* *to the signal game. Instead, it describes the interest rate action that minimizes the loss of (3.2.24) for any given combination* $\epsilon_j \times E\big(\epsilon_t \big| \epsilon_j\big) \times E\big[\epsilon_t \big| \epsilon_j\big]$.

We need now to determine what is the level of aggregate demand that obtains for any possible value of $\epsilon_j \times E\big(\epsilon_t \big| \epsilon_j\big)$ and of the shock ϵ_t. We do so substituting (3.2.25) into (3.2.21), obtaining:

$$2y\Big(\epsilon_t, \epsilon_j, E\Big[\epsilon_t \Big| \epsilon_j\Big]\Big) = \hat{y} + \epsilon_t - \frac{\phi^2}{\phi^2 + \psi}\left(\hat{y}(1-k) + \mu(\rho)\epsilon_j\right) + \frac{\psi}{\phi^2 + \psi}\sigma\Big(\mu(\rho)E\Big[\epsilon_t \Big| \epsilon_j\Big]\Big)^{a2}; \tag{3.2.26}$$

We now substitute (3.2.25) and (3.2.26) in the loss function of (3.2.24) to derive the indirect loss function for the Central Bank as a function of ϵ_t, $E\Big[\epsilon_t \Big| \epsilon_j\Big]$ and ϵ_t.

$$\begin{aligned} &L\Big(\epsilon_t, \epsilon_j, E\big[\epsilon_t \big| \epsilon_j\big]\Big) = \\ &= \left[\hat{y}(1-k) + \mu(\rho)\epsilon_t - \frac{\phi^2}{\phi^2 + \psi}(\hat{y}(1-k) + \mu(\rho)\epsilon_j) + \frac{\psi}{\phi^2 + \psi}\sigma\Big(\mu(\rho)E\big[\epsilon_t \big| \epsilon_j\big]\Big)^{a2}\right]^2 \\ &+ \psi\left[-\frac{\phi}{\phi^2 + \psi}\Big(\hat{y}(1-k) + \mu(\rho)\epsilon_j + \sigma\big(\mu(\rho)E\big[\epsilon_t \big| \epsilon_j\big]\big)^{a2}\Big)\right]^2; \end{aligned} \tag{3.2.27}$$

Note that the magnitude of the parameter $a1$, governing the responsiveness of money creation to a change in interest rates, does not enter into the loss function. Instead, $a1$ merely governs how responsive interest rates are to ϵ_t, ϵ_j and $E\big[\epsilon_t \big| \epsilon_j\big]$.

The indirect loss function of (3.2.27) is an essential building block for the solution of the signaling game to be studied in the next sections of the chapter. In fact, loosely speaking at this stage, the Central Bank can employ (3.2.27) to evaluate the payoff of various strategies for a given level of beliefs $E\big[\epsilon_t \big| \epsilon_j\big]$ held by agents. However, the solution of the signaling game needs to take into account that such belief $E\big[\epsilon_t \big| \epsilon_j\big]$ is itself a function of monetary policy. But interest rates are a function of ϵ_j. Therefore, (3.2.27) just maps any possible set of monetary policy actions, agent's beliefs $E\big[\epsilon_t \big| \epsilon_j\big]$ and shocks to output to the appropriate value for the loss function. The consistency of

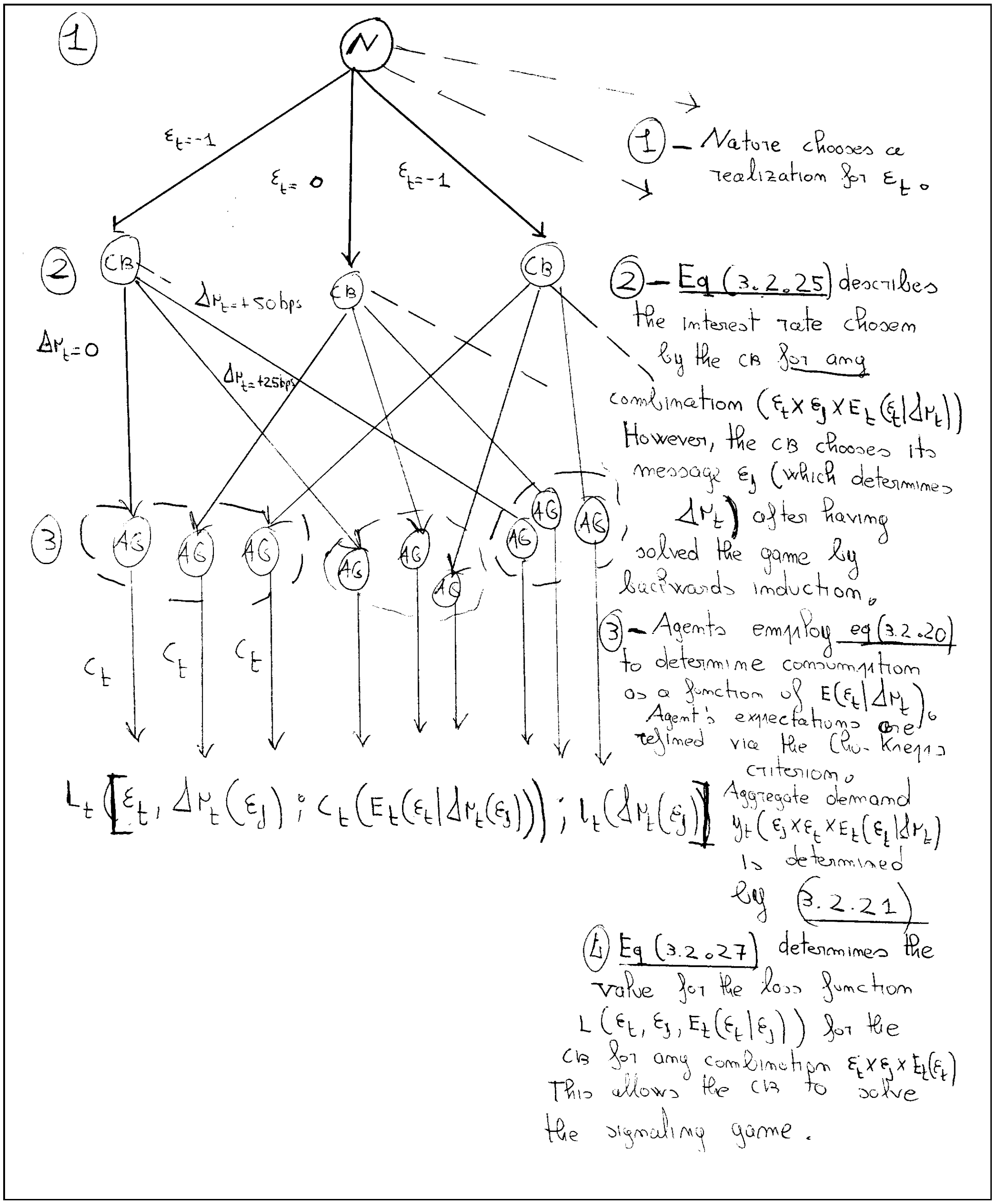

Figure 3.2: Linking the Building Blocks Together

agent's beliefs with the behavior of the Central Bank is to be analyzed when solving the signaling game.

It is useful, at this stage, to summarize how the various *building blocks* of the model fit together. For concreteness, a graphical representation is given in Figure 3.2.

Table 3.2 shows that nature makes the first move by choosing a realization for ϵ_t. The Central Bank, after observing ϵ_t, chooses its actions in *types space* by sending a message that it is type ϵ_j to the auctioneer. Equation (3.2.25) derives the monetary policy action undertaken by the Central Bank *for any given possible combination* of a message ϵ_j, a certain value for agents' beliefs $E\left[\epsilon_t \middle| \epsilon_j\right]$ and ϵ_t.

Aggregate consumption is chosen optimally after agents use the Central Bank's signal to extract information on *what information set tree they stand in.* The optimal choice of consumption as a function of *any given* $(\epsilon_t \times E\left[\epsilon_t \middle| \epsilon_j\right])$ is given by equation (3.2.19) and (3.2.20). Aggregate demand is then derived in equation (3.2.26) and aggregating over its various components and using (3.2.19) to determine consumption.

Hence, equation (3.2.27) delivers an indirect loss function for the Central Bank for *each possible combination of* $E_t\left[\epsilon_t \middle| \epsilon_j\right]$, message ϵ_j and type ϵ_t. In this way (3.2.27) can be used by the Central Bank to determine its optimal monetary for any given level of agents' beliefs $E\left[\epsilon_t \middle| \epsilon_j\right]$ (which are in turn conditional upon monetary policy).

A formal definition of the procedure adopted to solve the signaling game is given in Section 3.3.2.1. However, we might at this stage attempt to preview the intuition behind the solution concept. Note agents use (3.2.27) to determine what beliefs $E\left[\epsilon_t \middle| \epsilon_j\right]$ to hold conditional upon some appropriate refinement criterion to be later described to ensure that their beliefs are consistent with monetary policy. Hence, the Central Bank when solving the signaling game calculates by backwards induction what belief $E\left[\epsilon_t \middle| \epsilon_j\right]$ is associated to any particular action ϵ_j. This process, as will become clearer when we solve the signaling game, then allows the Central Bank to compute via (3.2.27) any loss function value for each type and for any combination of ϵ_j and $E\left[\epsilon_t \middle| \epsilon_j\right]$ that are compatible with agents' rational beliefs formation behavior and optimization process. Loosely speaking at this stage, the Central Bank in this way can compare the welfare impact of all its possible available strategies given that agents form beliefs rationally and then chooses its optimal one.

3.2.3.1 A Sanity Check: The Solution to the Model Under Perfect Information

We can perform an interesting sanity check for our analysis by studying the very special case of perfect information. Under perfect information, agents enjoy full knowledge of the magnitude of the shock to cash flows independently of the signal they receive from monetary policy. This implies that under perfect information $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t$.

In this framework, the Central Bank does not embark on a signaling game with agents and has no incentive not to reveal ϵ_t. We show that it is optimal for the Central Bank to set $\epsilon_t = \epsilon_j$ $\forall t$ by employing (3.2.27). so that the Central Bank always optimizes by revealing her true type under perfect information. This is a trivial result but it serves to check that our model is correctly specified.

To illustrate this result, we substitute for $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t$ into (3.2.27) and then show that the optimal choice for the Central Bank is to set $\epsilon_t = \epsilon_j$ since:

$$\left.\frac{\delta L\left(\epsilon_t, \epsilon_j, E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t\right)}{\delta \epsilon_j}\right|_{\epsilon_t = \epsilon_j} = \left\{ \left(\hat{y}(1-k) + \mu(\rho)\epsilon_t\right)\left(\phi - \frac{\phi^3}{\phi^2 + \psi} - \frac{\phi\psi}{\phi^2 + \psi}\right) \right\}; \tag{3.2.28}$$

$$= 0 \quad \forall \epsilon_t;$$

Therefore, equation (3.2.28) confirms the intuitive insight that, under perfect information, the Central Bank shall always reveal her type and play a perfectly separating equilibrium. This sanity check helps confirming that (3.2.27) is correctly derived.

Note, however, that this simple solution technique can be employed only to study the perfect information case since whenever the analysis is extended to imperfect information agents' beliefs $E\left[\epsilon_t \middle| \epsilon_j\right]$ cannot be taken to be fixed in equilibrium and are a function of the message ϵ_j.

Having setup the framework for the analysis, we start studying the solution of the signaling model before drawing some possible macroeconomic interpretations for our model.

3.3 Solving the Signaling Game

We investigate in this section a procedure to solve the signaling game. We proceed in two steps. First, we present in Section 3.3.1 a simple example of the solution of the signaling game when the shock ϵ_t follows a tri-nomial distribution so that there are only three possible types for the Central Bank.

However, this setting is not rich enough for our purposes. We therefore discuss in Section 3.3.2 a general method to analyze the signaling game when there are eleven possible types for the Central Bank.

The results of this section pave the way for the simulations we carry out in Section 3.4 in which we study various properties of the model.

3.3.1 A Simple Example

What is the macroeconomic consequence of the microeconomic fact that monetary policy can convey information to agents as to their expected capital income so that loosening (tightening) monetary policy can diminish (rise) consumer confidence according to the mechanism derived in (3.2.19)? We aim in this section to start exploring this question with a simple example. Rather than aiming to solve the model in the general case, we limit ourselves to start developing an intuition as to under what circumstances the microeconomic fact that monetary policy might signal to agents that they ought to revise their expected disposable income leads to the macroeconomic implication that the Central Bank is reluctant to use monetary policy aggressively to lean against the wind of shocks to agents' disposable income.

In the example we now develop we characterize two effects at work in the model. On the one hand, the *pooling effect* gives an incentive to the Central Bank not to implement an aggressive counter-cyclical monetary policy. In fact, equation (3.2.19) shows, consumers' confidence and hence aggregate consumers' spending might react negatively (positively) to a loosening (tightening) of monetary policy since agents employ the actions of the Central Bank as a signal to the information held by the Central Banker.

On the other hand, the *separating effect* might induce the Central Banker to reveal her type and implement counter-cyclical monetary policy as to stimulate (depress) investment in the face of a negative (positive) pattern of aggregate demand fluctuation via counter-cyclical monetary policy.

Note that the dichotomy that the *separating effect* acts only through investment and the *pooling effect* operates only via consumption is an over-simplistic artifact. In practice, the cost of borrowing affects consumers' spending, and not only investment, while investment also depends upon agents' expectations as to the magnitude of any shock. However, we adopt this simplification to make the signaling model tractable.

In this example, we let the shock to agents' cash flows ϵ_t take one of three a priori equi-probable values: i) one-third of times $\epsilon_t = 0$ and no shock occurs; ii) with an *ex ante* probability of one-third the economy is in recession since $\epsilon_t = -1$; iii) with a probability of one-third the economy is over-heating as $\epsilon_t = 1$.

Similarly, in this example we restrict the Central Bank to choose one of three possible strategies: a) it can keep rates on hold setting $\epsilon_j = 0$; b) it can hike rates, setting $\epsilon_j = 1$, which translates into a change in interest rates which magnitude is computed employing (3.2.25); c) finally, the Central Bank can loosen monetary policy by setting $\epsilon_j = -1$ in (3.2.25).

In this example as throughout the analysis, we set $k = 1$ in (3.2.24) so that the Central Bank is assumed to attempt to stabilize aggregate demand around its ex-ante expected level. We also set $\rho \approx 1$ so that $\mu(\rho) = 1$. We also set in (3.2.27) $\psi = 1$ and $a2 = 0.8$.

We study under which conditions the Central Bank plays a separating equilibrium. To fix ideas, we introduce the following important remark:

Remark 3.3.1. ***(Pure Separating Equilibrium under Imperfect Information Equivalent to the Perfect Information Outcome):*** *The separating equilibrium under the game of imperfect information has an important intuitive characterization. In fact, the Central Bank sets monetary policy as it would under perfect and symmetric information by agents under a separating equilibrium in the game of imperfect information in which agents are unsure as to the magnitude of the shock hitting cash flows .*

Proof. To understand the rationale behind the remark, consider the two properties characterizing a separating equilibrium: i) in a separating equilibrium $\epsilon_j = \epsilon_t$ $\forall t$ as the

Central Bank has no incentive to conceal its type from agents; ii) $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t$ as agents use Bayes's rule in a separating equilibrium to deduce the type of the Central Bank.

Note, however, that equation (3.2.28) proves that under perfect information the Central Bank sets $\epsilon_j = \epsilon_t \ \forall t$ as agents enjoy complete information so that $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t$ by definition. Hence, the conduct of monetary policy in a game of imperfect information characterized by a perfectly separating equilibrium and the manner in which the Central Bank sets rates under symmetric and complete information by agents are equivalent. $\square$

We now study under which conditions the setting assumed in this example delivers a pure separating equilibrium in which the *separation effect* dominates. We therefore need to investigate under what condition each possible type ϵ_t for the Central Bank finds it incentive compatible to set $\epsilon_t = \epsilon_j$ given that in a purely separating equilibrium $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t$.

We first show that a separating equilibrium is always incentive compatible for $\epsilon_t = 0$ given the coefficients for the parameters assumed in this example. In fact, (3.2.27) evaluated in a separating equilibrium implies that:

$$L\left[\epsilon_t = 0; \epsilon_j = 0; E\left[\epsilon_t \middle| \epsilon_j\right] = 0; \sigma; \phi; \psi; k = 1; a2 = 0.8\right] = 0; \tag{3.3.1}$$

Hence, type ϵ_t never wishes to deviate from the separating equilibrium. This is so for the assumption of $k = 1$ implies that the separating equilibrium outcome delivers the smallest possible value for the loss function of (3.2.24) attainable for type ϵ_t.

We now need to check under what conditions the separating equilibrium is incentive compatible for type $\epsilon_t = 1$. The first incentive compatibility constraint requires that type $\epsilon_t = 1$ does not wish to deviate from the separating equilibrium by setting $\epsilon_j = 0$ and pretending to be type $\epsilon_t = 0$ so that:

$$\begin{aligned} &L\left[\epsilon_t = 1; \epsilon_j = 1; E\left[\epsilon_t \middle| \epsilon_j\right] = 1; \sigma; \phi; \psi; k = 1; a2 = 0.8\right] \leq \\ &\leq L\left[\epsilon_t = 1; \epsilon_j = 0; E\left[\epsilon_t \middle| \epsilon_j\right] = 0; \sigma; \phi; \psi; k = 1; a2 = 0.8\right]; \end{aligned} \tag{3.3.2}$$

This condition, given our choice of parametrization, is satisfied if and only if:

$$\frac{(1+\sigma)^2}{\phi^2 + 1} \leq 1; \tag{3.3.3}$$

This equation confirms the intuition that the Central Bank is likely to play a separating strategy when: i) ϕ is relatively high and hence the transmission mechanism has a relatively powerful counter-cyclical effect via the borrowing-cost channel (embodied by what we termed the *separating effect*); ii) while σ is low enough for a low weight to be attached to wealth effects and consumers' confidence in the determination of aggregate consumption in (3.2.14) (so that the *separation effect* is relatively weak).

We now study under what conditions type $\epsilon_t = 1$ deviates from the separating equilibrium by setting $\epsilon_j = -1$ so that $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_j = -1$. For this type not to deviate from the separating equilibrium the following incentive compatibility constraint must hold:

$$\begin{aligned} &L\Big[\epsilon_t = 1; \epsilon_j = 1; E\left[\epsilon_t \middle| \epsilon_j\right] = 1; \sigma; \phi; \psi; k = 1; a2 = 0.8\Big] \leq \\ &\leq L\Big[\epsilon_t = 1; \epsilon_j = -1; E\left[\epsilon_t \middle| \epsilon_j\right] = -1; \sigma; \phi; \psi; k = 1; a2 = 0.8\Big]; \end{aligned} \tag{3.3.4}$$

Employing (3.2.27) given our choice of parametrization, this incentive compatibility condition implies that:

$$\sigma \leq +2 + \sqrt{4 + 8\phi^2}; \tag{3.3.5}$$

Note that whenever (3.3.3) holds, then numerical analysis shows that also (3.3.5) holds.

We now need to investigate the incentive compatibility conditions for the case that $\epsilon_t = -1$. Note, however, an important property of symmetry of (3.2.27):

$$\begin{aligned} &L\Big[\epsilon_t = c_1; \epsilon_j = c_2; E\left[\epsilon_t \middle| \epsilon_j\right] = c_3; \sigma; \phi; \psi; k = 1; a2 = 0.8\Big] = \\ &= L\Big[-\epsilon_t = c_1; -\epsilon_j = c_2; -E\left[\epsilon_t \middle| \epsilon_j\right] = c_3; \sigma; \phi; \psi; k = 1; a2 = 0.8\Big]; \quad \forall \phi, \sigma, \psi, a2; \forall (c_0, c_2, c_3) \epsilon \Re; \end{aligned} \tag{3.3.6}$$

This property implies that the indirect loss function of (3.2.27) is symmetric around zero whenever $k = 1$. This is so for $k = 1$ renders (3.2.24) also symmetric around zero. Hence, for illustration, for the Central Bank to witness that aggregate demand is above its trend level by 1 % and inflation stands at 2 % has the same welfare implication as observing aggregate demand below trend by 1 % and deflation at a rate of 2%. The property of symmetry of equation (3.3.6) implies that the incentive compatibility condition for type $\epsilon_t = 1$ are the same as for type $\epsilon_t = -1$.

This observations imply the following conclusive remark:

Remark 3.3.2. ***(Insight of the Simple Example:)*** *When the distribution of ϵ_t follows the tri-nomial distribution assumed in the example of this section together with the parametrization $k = \rho = \psi = 1$ and $a2 = 0.8$ for (3.2.27), the pure separating equilibrium under imperfect information (analogous to the outcome of the model under complete information as shown by the remark of (3.3.1)) does not unravel as long as this single binding constraint holds:*

$$\frac{(1+\sigma)^2}{\phi^2+1} \leq 1; \sigma \leq 1$$

The insight provided by the example is worth re-iterating. The Central Bank behaves under the game of asymmetric information as it would under complete and symmetric information if, and only if, ϕ is large relative to σ. To interpret this condition, it must be borne in mind that two effects link monetary policy to aggregate demand. On the one hand, a traditional component of the transmission mechanism is at work by which aggregate demand is diminishing in the cost of borrowing. The larger is ϕ (the responsiveness of investment to monetary policy), the greater weight is carried by such effect.

On the other hand, the realization that counter-cyclical monetary policy might trigger off pro-cyclical wealth effects induces the Central Bank to consider an unusual effect of the transmission mechanism, which we termed the *pooling effect*. This effect biases the Central Bank towards inertia in this example by giving to the Central Bank an incentive not to reveal its type to the public. This *pooling effect* is the more powerful the larger is σ, the weight attached to wealth effects in the determination of aggregate consumption in (3.2.14).

When ϕ is large relative to σ, the traditional view of the transmission mechanism dominates over the *pooling effect* and hence the Central Bank finds it optimal to embark into aggressive counter-cyclical monetary policy rather than trying to prevent panic and pro-cyclical wealth effects by undertaking an inactive monetary policy stance.

This insight clarifies future results. However, we now proceed to generalize the setting of this example to a richer scenario.

3.3.2 A General Solution Concept

This section plays a double duty. Its first purpose consists of specifying what assumptions are applied to the general scenario we analyze and what game-theoretic characteristics the solution must satisfy. This task is undertaken in Section 3.3.2.1. Secondly, we aim to list a number of useful properties of the model that greatly simplify the simulation analysis. We illustrate such list of expedients in Section 3.3.2.2.

3.3.2.1 The Solution Concept: A Bayesian Equilibrium subject to Cho-Kreps Refinement

We first of all need to specify the distribution for the shock to agents' cash flows ϵ_t:

Assumption 3.3.1. ***(Distribution of ϵ_t):*** *The shock ϵ_t is assumed to be an integer number and takes one of eleven equi-probable values drawn from an independent uniform distribution:*

$$\epsilon_t \sim UIN[-5,5] \quad \epsilon_t = -5, 4, 3..0..3, 4, 5; \tag{3.3.7}$$

We say that the economy is in recession regime if $\epsilon_t < 0$ so that a negative shock to agents' disposable income has occurred; conversely, we define the overheating regime as one in which $\epsilon_t > 0$ so that a disposable-income enhancing shock has occurred.

We do not impose any restriction on the signal ϵ_j sent by the Central Bank and on the interest rate Δr_t.

This setting is rich enough to deliver a number of different properties. However, first we need to define the properties of the Perfect Bayesian Equilibrium solution concept adopted to solve the game of imperfect information. We refine such solution by imposing the Cho-Kreps intuitive criterion (Cho and Kreps 1987). We provide a formal definition of the solution criterion adopted by following the discussion in Fudenberg and Tirole ((Fudenberg and Tirole 1991), ch.6 and ch.8).

Definition 3.3.1. ***(Perfect Bayesian Equilibrium with Cho-Kreps refinement):*** *A Perfect Bayesian Equilibrium refined through the Cho-Kreps intuitive criterion of the* *signaling game, whose payoff for the sender is summarized in (3.2.27), is a strategy profile consisting of: a set of optimal signals by the Central Bank denoted as ϵ_j^*, each triggering off interest rate changes according to (3.2.25); a set of consumption profiles by the representative agent c_t^* endowed with a quadratic utility function in consumption denoted as U(.) which is optimized by the consumption plans of (3.2.19) given ex-ante beliefs p(.) on the distribution of ϵ_t and posterior beliefs $E\left[\epsilon_t \middle| \epsilon_j\right]$ such that properties (P1) to (P4) hold:*

- *(P1):* $\forall \epsilon_t, \quad \epsilon_j * \in \ arg \ \min_{\epsilon_j} L\Big(\epsilon_j, c_t(E\left[\epsilon_t \middle| \epsilon_j\right])\Big|\epsilon_t\Big)$ *;*
- *(P2):* $\forall c_t, \quad c_t * \in \ arg \ \max_{c_t} U\Big(\epsilon_t, c_t(E\left[\epsilon_t \middle| \epsilon_j\right])\Big)$ *;*
- *(P3):* $E\left[\epsilon_t \middle| \epsilon_j\right] = \sum_1^n p\left(\epsilon_t \middle| \epsilon_j\right) \epsilon_t \ st. \ \epsilon_j \in arg \ \min_{\epsilon_j} L\Big(\epsilon_j, c_t(E\left[\epsilon_t \middle| \epsilon_j\right])\Big|\epsilon_t\Big)$
  $\forall \epsilon_j \ s.t \ \epsilon_j = \epsilon_j * \ \ for \ some \ \epsilon_t \in (-5, 5)$
- *(P4): Beliefs off the Equilibrium Path must be subject to the Cho-Kreps Intuitive Criterion. If $\hat{\epsilon_j}$ lies off the equilibrium path, the receiver must believe that type $\hat{\epsilon_t}$ never plays $\hat{\epsilon_j}$ so that $p(\hat{\epsilon_t}|\hat{\epsilon_j}) = 0$ whenever $L(\epsilon_t, \overline{\epsilon_j}) < L(\epsilon_t, \hat{\epsilon_j})$ for any strategy $\overline{\epsilon_j}$ other than $\hat{\epsilon_j}$ given the receiver's beliefs profile.*

We now proceed to explain and interpret each of the four conditions in turn.

First of all, (P1) states that, taking a given profile for consumers' expectations $E\left[\epsilon_t \middle| \epsilon_j\right]$ for each ϵ_j, the Central Bank's strategy ϵ_j must be optimal so that it minimizes (3.2.27) for *any possible combination*$(\epsilon_j \times E\left[\epsilon_t \middle| \epsilon_j\right] \times \epsilon_t)$. It is worth re-iterating that (3.2.25) translates any choice of ϵ_j into a given choice for Δr_t.

The second condition of (P2) simply states that, given any set of ex-post beliefs $E\left[\epsilon_t \middle| \epsilon_j\right]$ for each ϵ_j, receivers must optimize their payoff function by setting consumption according to the optimal rule of (3.2.19).

The other two conditions aim to impose some restrictions on how agents form beliefs, so that the process of beliefs formation is, in some sense, rational. (P3) defines how agents form beliefs along the equilibrium path. In fact, the condition imposes the restriction that beliefs must be calculated along the equilibrium path by using Bayes rule. If signal $\hat{\epsilon_j}$ is sent by the Central Bank, the receivers first use equilibrium conditions to understand what types ϵ_t play $\hat{\epsilon_j}$ in equilibrium according to (P1). Then, $E\left(\epsilon_t \middle| \hat{\epsilon_j}\right)$ is computed by taking the average value (since the ex-ante distribution is uniform no weighting is necessary) of ϵ_t for all the types playing $\hat{\epsilon_j}$ according to (P1).

Finally, a restriction is imposed upon beliefs in the off equilibrium path by (P4) according to the intuitive criterion first developed by Cho and Kreps (Cho and Kreps 1987). The requirement we have formally stated in (P4) is best illustrated by a simple example.

Imagine, for pure illustration, the existence of an equilibrium in which type $\epsilon_t = 0$ plays $\epsilon_j = 1$. Type $\epsilon_t = 0$ might consider playing $\epsilon_j = 0$, but the representative agent believes that only type $\epsilon_t = 5$ would play $\epsilon_j = 0$. Therefore if no interest rate change is announced and $\epsilon_j = 0$ is indeed played, then $E\left[\epsilon_t \middle| \epsilon_j = 0\right] = 5$ and type $\epsilon_t = 0$ may be indeed better off playing $\epsilon_j = 1$ avoiding to pool with $\epsilon_t = 5$ at an information set off the equilibrium path. Intuitively, this is so for the Central Bank risks sending (in a non-optimal manner) a very misleading signal to agents leading them to incorrectly believe that a large positive shock to their disposable income has occurred if type ϵ_t does not move interest rates and plays $\epsilon_j = 0$. This is not optimal and the refinement criterion implies that agents cannot believe that the Central Bank would play a non-optimal strategy off the equilibrium path.

In fact, the refinement criterion forces agents to ask themselves a further question: given equilibrium beliefs on the off-path information sets, would type $\epsilon_t = 5$ really play $\epsilon_j = 0$?

However, assume that type $\epsilon_t = 5$ in equilibrium is better off playing $\epsilon_t = \epsilon_j = 5$ rather than $\epsilon_t = 0$ for whatever belief agents may have on the economy given that $\epsilon_j = 0$ is played.

Therefore the equilibrium we have hypothesized, the refinement criterion states in this example, rests on the receiver believing that at some off equilibrium path the sender must be playing a strategy which makes the sender itself (the Central Bank when type $\epsilon_t = 5$) worse off given the current equilibrium of the game.

The belief $\epsilon_t = 5$ at $\epsilon_j =$ cannot be accepted and must be refined by the Cho-Kreps criterion. Once a new belief is created at the information set $\epsilon_j = 0$, it may be that type $\epsilon_t = 0$ may wish to reconsider her strategy given that it would face more favorable wealth effects when rates are on hold. This completes our intuitive account of the conditions characterizing a Perfect Bayesian Equilibrium refined with the Cho-Kreps criterion.

3.3.2.2 Some Useful Properties of the Model

This short section only aims to summarize some observations that greatly simplify the simulation analysis of Section 3.4.

We first remark that when $k = 1$ in (3.2.24), so that the Central Bank is assumed to aim to stabilize aggregate demand around its trend level, a remarkable symmetry property applies to (3.2.27):

Remark 3.3.3. ***(Symmetry Property for (3.2.27)):*** *The payoff function for the Central Bank of* 3.2.27 *is endowed with the following property of symmetry:*

$$L\Big(\epsilon_t; \hat{\epsilon_j}; E\big(\epsilon_t \big| \hat{\epsilon_j}\big); \sigma; \phi; \psi; k; a2\Big) = \\ = L\Big(- \epsilon_t; -\hat{\epsilon_j}; -E\big(\epsilon_t \big| \hat{\epsilon_j}\big); \sigma; \phi; \psi; k; a2\Big); \tag{3.3.8}$$

Proof. The result of Remark 3.3.3 is verified by evaluating the quadratic loss function of (3.2.27). □

Remark 3.3.3 greatly simplifies the analysis as we set in all simulationS $k = 1$. The remark implies that the incentive compatibility conditions in the Over-Heating Region are the mirror image of the ones in the Recession Area- hence we only need to consider the incentive compatibility constrains of six types rather than eleven.

Furthermore, this property of symmetry makes the analysis of pooling actions to $\epsilon_j = 0$ particularly simple. If type $\epsilon_t = 1$ wants to pool to $\epsilon_j = 0$ given beliefs $E[\epsilon_t|\epsilon_j = 0] = 0$, then also type $\epsilon_t = -1$ will find the strategy incentive compatible. And hence $E\big[\epsilon_t \big| \epsilon_j\big] = 0$ when no interest rate change is implemented since the Central Bank plays in this case $\epsilon_j = 0$. In fact, type $\epsilon_t = 0$ always plays $\epsilon_j = 0$ given that $E\Big[\epsilon_t \Big| \epsilon_j = 0\Big] = 0$. By the same token, if $\epsilon_t = -2$ prefers to play $\epsilon_t = 0$, then also type $\epsilon_t = -2$ opts to do so. And again beliefs will be $E\Big[\epsilon_t \Big| \epsilon_j = 0\Big] = 0$. The argument can be generalized in the following remark:

Remark 3.3.4. ***(Beliefs when Rates on Hold):*** *The symmetry property of (3.3.3) implies that* $E\Big[\epsilon_t \Big| \epsilon_j = 0\Big] = 0$ *so that, if interest rates are kept on hold, agents rationally believe that the expected value for the shock to cash flows* ϵ_t *is zero.*

Proof. Assume, in fact, the conclusion were false and $E\Big[\epsilon_t\Big|\epsilon_j = 0\Big] \neq 0$. It would then follow that some type in the recession regime for which $\epsilon_t = \hat{\epsilon}_t < 0$ has a different optimal strategy than the one played by type $\epsilon_t = -\hat{\epsilon}_t > 0$. However, this would contradict remark (3.3.3) which shows that type $\epsilon_t = \hat{\epsilon}_t$ faces the same incentive compatibility conditions as faced by type $\epsilon_t = -\hat{\epsilon}_t$.

Hence, denoting with ϵ_j^* an optimal strategy, if $\epsilon_j^* = \hat{\epsilon}_j$ for type $\hat{\epsilon}_t$, then it must be that $\epsilon_j^* = \hat{\epsilon}_j$ for type $-\hat{\epsilon}_t$. This proves the remark. □

This is also a very useful remark because implies that agents do not expect to be neither in the recession nor in the over-heating area when interest rates are kept on hold. Instead, when rates are kept on hold, Remark 3.3.4 states, agents find themselves unable to update their ex-ante beliefs by forming a view as to in what direction is their disposable income likely to depart from its average level. This remark makes the computation of the payoff for the strategy $\epsilon_j = 0$ particularly simple, as shown in the next remark:

Remark 3.3.5. ***(Welfare when rates on hold):***

The equilibrium payoff to the Central Bank not moving rates takes a particularly simple form whenever k =1:

$$L\Big(\epsilon_t; \epsilon_j = 0; E\big[\epsilon_t\big|\epsilon_j\big] = 0; \sigma; \phi; \psi; k = 1; a2\Big) = \epsilon_t^2 \quad \forall \epsilon_t, \sigma, \phi, \psi, a2; \tag{3.3.9}$$

Proof. Recall that Remark 3.3.4 implies that $E\Big[\epsilon_t\Big|\epsilon_j = 0\Big] = 0$. Substitute this together with $\epsilon_j = 0$ in (3.2.27) when k=1 to verify the remark. □

In fact, when no interest rate move is decided no wealth effect is engendered because agents cannot then use monetary policy to update their ex-ante belief on the magnitude of shocks to their wealth. Absent wealth effects, all parameters tied to the expectation term $E\big[\epsilon_t\big|\epsilon_j\big]$ in (3.2.27) become irrelevant. Given that rates are on hold, (3.2.23) implies that the price level is stable, which annihilates the effect of ψ. Hence, when interest rates do not move, the loss function depends solely on the square level of the shocks hitting output.

Bearing in mind these remarks, we can now proceed to simulate the model as to draw its macroeconomic implications.

3.4 Qualitative Implications for Monetary Policy

Under what conditions does the model imply that the Central Bank reacts timidly to a shock in macroeconomic fundamentals as not to trigger off pro-cyclical wealth effects? And what is the effect of publishing the minutes of the Interest Rate setting Panel and why should a Central Bank follow the FED's practice of not sharing its macroeconomic forecasts with the public? Moreover, can information secrecy be welfare rising? We aim to explore such questions in this section.

We study in Section 3.4.1 the implications of asymmetric information for monetary policy. We proceed in Section 3.4.2 by carrying out some simulations of the model. We then study in Section 3.4.3 whether information secrecy is on our model welfare optimal. We proceed in Section 3.4.4 to investigate the effect of mandating that the Central Bank should publish the minutes of the Interest Rate Setting Panel, and show that, under appropriate assumptions, such innovation tends to make interest rates change more frequently and by a greater magnitude relative to the secrecy scenario.

We also illustrate the effects of altering some of the parameters in the model in Section 3.4.4. We finally formulate a conjecture in Section 3.4.6 that the model can bias the ratio of *continuations* to *reversals* in monetary policy in favor of *continuations*.

3.4.1 Optimal Inertia and Gradualism: The Impact of the Informational Content of Interest Rates on Monetary Policy

The results of this section are based upon the simulation results presented in Section 3.4.2. However, for ease of exposition we prefer presenting the implications of the simulations before reporting some of the simulations results in Section 3.4.2.

Why does the microeconomic assumption that agents under asymmetric information extract their wealth expectations from the behavior of interest rates has the macroeconomic consequence of biasing monetary policy towards inactivity or gradual adjustment under asymmetric information relative to the symmetric information benchmark? How does the conduct of monetary policy vary as a function of σ, the parameter capturing the weight attached to wealth effects in equation (3.2.14)?

We explore these questions in this section. We illustrate the results in two steps.

First, we summarize the macroeconomic implications of the simulation work in Section 3.4.1.1. Then, we present in Section 3.4.2 the results of some simulations relevant to this section.

3.4.1.1 The Implications of the Analysis

We try to show that the model analyzed in this chapter can contribute to one explanation as to why Central Banks act, in the definition of Goodhart (Goodhart 1997), *too little and too late*. Such claim often refers to the fact that Central Banks do not immediately react to the information acquired about the magnitude of shocks on macroeconomic fundamentals, and, in spite of a large shock to, for instance, aggregate demand, might decide to leave rates initially on hold or to embark in a policy of only gradual adjustment of monetary policy.

We show that the model suggests one possible reason as to why Central Banks find this policy of *inertia and gradualism* optimal rather than stemming from a policy mistake. We first fix ideas by defining two important terms to which we refer in the discussion:

Definition 3.4.1. ***(Inertia and Gradualism):*** *We define inertia as arising when interest rates are on hold in spite that the macroeconomic shock to agents' disposable income is of non-zero magnitude so that* $\epsilon_t \neq 0$ *but* $\Delta r_t = \epsilon_j = 0$.

We define gradualism as arising when interest rates move in the asymmetric information regime by a smaller magnitude that the model would imply under symmetric information.

An important implication of the model lies in the finding that the microeconomic fact that interest rates act to convey to agents information as to the magnitude of their wealth effects tends to bias monetary policy towards inertia and gradualism relative to the full information benchmark.

In fact, when observing that a negative (positive) shock to output has occurred, the Central Bank might be tempted to adopt a very aggressive approach and let interest rates be lower (higher) to stimulate (depress) investment demand. However, in so doing, the insight of the model holds, it can lead agents to rational panic (euphoria) as agents learn the information the Central Bank holds as to the likely evolution of their disposable income.

To avoid triggering off such pro-cyclical wealth effects, the Central Bank might decided to keep rates on hold (adopting *inertia*) or to move rates by a minimal amount (adopting gradualism) as opposed to the large jump in the level of interest rates the Central Bank might have effected under symmetric information.

Note that, in fact, if agents know the magnitude of the shock occurring to their cash flows without having to try to infer it by observing monetary policy, the Central Bank has no incentive for *gradualism or inertia* as in this case monetary policy does not risk triggering off any pro-cyclical wealth effect. We formalize such insights in the following proposition:

Proposition 3.4.1. ***(Asymmetric Information Leads to Inertia and Gradualism):***

Asymmetric Information on the magnitude of ϵ_t between agents and the Central Bank implies the properties of inertia and gradualism as: i) interest rates are left unchanged more often under asymmetric information than under symmetric information between agents and the Central Bank on the magnitude of ϵ_t; ii) Instead when interest rates are not kept on hold, the rate of change of interest rates for any type ϵ_t under asymmetric information is never higher than it is in the symmetric information regime.

Proof. We first prove the second part of the proposition. Recall that equation (3.2.28) proves that under symmetric information $\epsilon_t = \epsilon_j^* \; \forall t$ since a pure separating equilibrium is incentive compatible for the Central Bank when agents have complete information on the magnitude of the shock to cash flows ϵ_t. Denote the strategy played by type $\hat{\epsilon}_t$ under perfect information $\epsilon_j^{sy}(\hat{\epsilon}_t)$.

Denote with $\epsilon_j^{asy}(\hat{\epsilon}_t)$ the strategy played by type $(\hat{\epsilon}_t)$ under asymmetric information. The simulation results of Section 3.4.2 show that:

$$\epsilon_j^{asy}(\hat{\epsilon}_t) = \begin{cases} \leq \epsilon_j^{sy}(\hat{\epsilon}_t) \quad iff \quad \hat{\epsilon}_t > 0; \\ \geq \epsilon_j^{sy}(\hat{\epsilon}_t) \quad iff \quad \hat{\epsilon}_t < 0; \; ; \\ = \epsilon_j^{sy}(\hat{\epsilon}_t) \quad iff \quad \hat{\epsilon}_t = 0; \end{cases} \tag{3.4.1}$$

Employing the results of equation (3.4.1) into equation (3.2.25) and denoting with Δr_t^{sy} and Δr_t^{asy} monetary policy under symmetric and asymmetric information respectively, yields:

$$\Delta r_t^{asy}(\hat{\epsilon}_t) = \begin{cases} \leq \Delta r_j^{sy}(\hat{\epsilon}_t) & iff \quad \hat{\epsilon}_t > 0; \\ \geq \Delta r_j^{sy}(\hat{\epsilon}_t) & iff \quad \hat{\epsilon}_t < 0; \\ = \Delta r_j^{sy}(\hat{\epsilon}_t) & iff \quad \hat{\epsilon}_t = 0; \end{cases} ; \tag{3.4.2}$$

This proves the second part of the proposition. To prove the first part of the proposition, note that while $\epsilon_j^{sy}(\hat{\epsilon}_t) = \hat{\epsilon}_t \ \forall t$, simulations in Section 3.4.2 show that for some types under asymmetric information $0 = \epsilon_j^{sy}(\hat{\epsilon}_t) \neq \hat{\epsilon}_t \ \forall t$, so that interest rates are more likely to be on hold under symmetric information. $\square$

Proposition 3.4.1 then shows why under asymmetric information the conduct of monetary policy by the Central Bank is biased towards *inertia and gradualism.* However, the magnitude of this bias tends to be increasing in the weight σ given to $E\left[\epsilon_t \middle| \epsilon_j\right]$ in (3.2.27).

To see why this is so consider the benchmark case in which $\sigma = 0$ and domestic agents do not own domestic equities. In such case, monetary policy is not informative as to the optimal consumption plan to be adopted. In fact, agents are assumed to know their labor income with certainty. Therefore, they need to revise through inference based on monetary policy only their expected capital income. However, when σ is zero agents' consumption plans are insensitive to equity returns and hence, in the restrictive setting of the model, monetary policy does not risk triggering off any pro-cyclical wealth effect.

Instead, as σ rises, the Central Bank gets increasingly more cautious about employing counter-cyclical monetary policy aggressively as this risks triggering off some large pro-cyclical wealth effects. We formalize such insight in the following Proposition:

Proposition 3.4.2. ***(Gradualism and Inertia Rising in the Informativeness of Interest Rate Changes):*** *Intertia and Gradualism are rising in the weight σ agents place on their capital income when determining optimal consumption plans via (3.2.14).*

Proof. The simulations result show that the difference between $\epsilon_j^{asy}(\hat{\epsilon}_t)$ and $\epsilon_j^{sy}(\hat{\epsilon}_t)$ is rising in σ. Note also that when $\sigma = 0$, $\epsilon_j^{asy}(\hat{\epsilon}_t) = \epsilon_j^{sy}(\hat{\epsilon}_t)$ and hence the Central Bank behaves as under perfect information without facing any incentive for inertia or gradualism. $\square$

The analysis of the signaling game also yields some very interesting limit pricing results, as can be observed by Table 3.3. We prove and define this limit pricing results in Proposition 3.4.3.

However, a simple intuitive account for such limit pricing strategy can be provided before we proceed to formalize the result. Imagine the Central Bank has observed a mildly recessionary shock (so that $\epsilon_t = -4$) and might want to decrease interest rates by fifty basis points. However, the Central Bank fears that if it does so, agents might believe that it has in fact observed a very large recessionary shock. This is so for a fifty basis points move would also be implemented by the Central Bank when it observes that a very severe recession might be happening ($\epsilon_t = -5$) so that the Central Bank might opt in this case to the type $\epsilon_t = -4$. In other words, playing a strategy of $\epsilon_j = -4$ does not bring about agents beliefs to be $E\left(\epsilon_j \middle| \epsilon_t = -4\right) = -4$ since agents would believe that type $\epsilon_t = -5$ would also play $\epsilon_j = -4$.

Hence, the Central Bank, might decide to lower rates only by less than fifty basis points as to avoid inducing agents to believe that it might have observed a very large negative shock to their disposable income of magnitude $\epsilon_t = -5$. Having stated the result of limit pricing informally, we now proceed to formalize it:

Proposition 3.4.3. *(**Limit Pricing Effect**:) Even if type $\hat{\epsilon}_t$ plays a separating strategy, it might still set under asymmetric information $\epsilon_j^{asy}(\hat{\epsilon}_t) \neq \hat{\epsilon}_t$ as to prevent other types from pooling to its strategy.*

Hence, interest rate changes under asymmetric information can differ from the perfect information setting even for those types playing a separating strategy.

Proof. Follows from results of Section 3.4.2. See in particular the results of the simulation carried out in Section 3.4.2.4. □

We now present some simulation results that back some of the statement made when proving the findings of this section.

3.4.2 The Simulation Results

The aim of this section lies in describing the results of some of the simulations of the model we have carried out to illustrate our qualitative results. The implications of the results of this section have been previously drawn in Section 3.4. Therefore, we limit ourselves in this section to describing concisely the results of some of the simulations carried out, which are fully derived in Appendix B.1.

We start by reporting in this section the results obtained by altering across different scenarios the magnitude of the parameter σ in the model, which governs the importance of the term $E\left[\epsilon_t \middle| \epsilon_j\right]$ in determining aggregate demand in (3.2.21). We derive a Perfect Bayesian Equilibrium refined with the Cho-Kreps intuitive criterion following the equilibrium requirements described in Section 3.3.2.1.

We fix the other parameters throughout the simulations of this section to take this constellation of values: $\phi = \rho = \psi = 1, a2 = 0.8$. We experiment simulating the model modifying such parameters in Section 3.4.5.

3.4.2.1 Simulation Results when $\sigma = 0.8$

We start the exercise by fixing parameters at the following level: $\sigma = 0.8, \phi = \psi = k = 1, a2 = 0.8$. This constellation of parameters implies that interest rates are always on hold, as shown by Table 3.1 summarizing results which we formally derive in Section B.1.0.1 in the appendix. The intuition for the results lies in the fact that in this case σ is high which implies widespread stock-ownership. Hence the Central Bank finds it very costly to implement counter-cyclical monetary policy and trigger off pro-cyclical wealth effects. This a pretty degenerate and extreme case, but it represents one of the polar cases to which the simulation results can give rise.

3.4.2.2 Simulation Results when $\sigma = 0.53$

As we now let $\sigma = 0.53$, the informational content of interest rates decreases relative to the previous simulation case; wealth effects become less significant and hence the Central Bank triggers off smaller pro-cyclical wealth effects by using monetary policy aggressively.

It is quite interesting to notice that the simulation results illustrated by Table 3.2 deliver five possible outcomes for monetary policy: rates can stay on hold, move in either direction by a small amount or be modified in either directions by a large amount. This mirrors, if only at a qualitative level, the practice followed by most OECD Central Banks.

The outcome for this simulation exercise depicted in Table 3.2 shows that if the shocks observed by the Central Bank fall below a certain threshold value, interest rates stay on hold. Hence seven types out of eleven pool to type $\epsilon_t = 0$ by playing strategy $\epsilon_j = 0$, implementing hence hence a semi-pooling strategy.

Outcome of the signaling Game: Perfect Pooling

Type $\epsilon_{\mathbf{t}}$	**Strategy** ϵ_j	$(\Delta r)^{a1}$	**Beliefs** $E[\epsilon_t \vert \epsilon_j]$
0	0	0	0
-1,1	0	0	0
-2,2	0	0	0
-3,3	0	0	0
-4,4	0	0	0
-5,5	0	0	0
Off Path	$0 \prec \epsilon_t \prec 1$		1
	$1 \prec \epsilon_t \prec 2$		2
	$2 \prec \epsilon_t \prec 3$		3
	$3 \prec \epsilon_t \prec 4$		4
	$4 \prec \epsilon_t \prec 5$		5

Table 3.1: Monetary Policy when $\sigma = 0.8; \phi = \psi = k = 1; a2 = 0.8$

Outcome of the Signaling Game: Symmetry with Five Regimes

Type $\epsilon_{\mathbf{t}}$	**Strategy** ϵ_j	$(\Delta r)^{a1}$	**Beliefs** $E[\epsilon_t \vert \epsilon_j]$
0	0	0	
-1,1	0	0	
-2,2	0	0	
-3,3	0	0	
-4,4	-3.54,3.54	-2.49,+2.49	
-5,5	-5,5	-3.46,+3.46	
Off Path	$0 \prec \epsilon_t \prec 1$		1
	$1 \prec \epsilon_t \prec 2$		2
	$2 \prec \epsilon_t \prec 3$		3
	$3 \prec \epsilon_t \prec 3.54$		4
	$3.54 \prec \epsilon_t \prec 4$		4.5
	$4 \prec \epsilon_t \prec 5$		5

Table 3.2: Monetary Policy when $\sigma = 0.53; \phi = \psi = k = 1; a2 = 0.8$

Instead, if large output shocks occur, counter-cyclical monetary policy is implemented as the Central Bank plays in this case a separating strategy letting agents learn the magnitude of the shock that has occurred to the economy. In this case the Central Bank opts to trigger off a counter-cyclical change in investment even if this implies that it has to reveal its type to agents that learn the asymmetric information the Central Bank is endowed with.

A step by step formal derivation of the results of this simulation exercise is given in Section B.1.0.2 in the appendix.

3.4.2.3 Simulation Result when $\sigma = 0.4$

We now let the informational content of interest rate changes drop even further as $\sigma = 0.4$. The outcome of the game is depicted in Table 3.3, which shows that now the signaling game delivers a pure separating outcome: the Central Bank reveals its type to agents since wealth effects are not strong enough for the Central Bank to have an incentive to play a pooling strategy.

Note, however, that the very interesting property of *limit pricing* holds. In fact, note that type $\epsilon_t = 4$, for instance, in spite of playing a pure separating strategy opts to play $\epsilon_j = 3.84 < 4$, as shown by Table 3.3. This is so for type $\epsilon_t = 4$ knows that if she plays $\epsilon_j = 4$, it then provides also to type $\epsilon_t = 5$ an incentive to also play $\epsilon_j = 4$. Hence under the Cho-Kreps refinement criterion agents cannot rationally believe that $E\left(\epsilon_t \middle| \epsilon_j = 4\right) = 4$ since to set $\epsilon_j = 4$ is not optimal for type $\epsilon_t = 4$ given that also type $\epsilon_t = 5$ would do the same. Then type $\epsilon_t = 4$ to ensure that it differentiates itself from type $\epsilon_t = 5$ plays a limit pricing strategy. The results of this simulation are formally derived in Section B.1.0.3 in the appendix.

3.4.2.4 Simulation Results when $\sigma = 0.2$

We not study the polar case of perfect separation without limit pricing. If $\sigma = 0.2$, wealth effects are so low that the Central Bank does not find it beneficial to try to conceal its type from agents. It does not even engage into limit pricing, as shown by the results in Table 3.4. Therefore, the outcome of this section is analogous to the one that applies to the symmetric information regime. The results of this simulation exercise are formally

Outcome of the signaling Game: Separation with Limit Pricing

Type $\epsilon_\mathbf{t}$	**Strategy** ϵ_j	$(\Delta r)^{a1}$	**Beliefs** $E[\epsilon_t\|\epsilon_j]$
0	0	0	0
-1,1	-1, 1	0.77	-1,1
-2,2	-1.97, 1.97	-1.34,+1.34	-2,2
-3,3	-2.82, 2.82	-1.98,1.98	-3,3
-4,4	-3.78,3.78	-2.6,+2.6	-4,4
-5,5	-5,5	-3.22,3.22	-5,5
Off Path	$0 \prec \epsilon_t \prec 1$		1
	$1 \prec \epsilon_t \prec 1.97$		2
	$1.97 \prec \epsilon_t \prec 2$		2.5
	$2 \prec \epsilon_t \prec 2.82$		3
	$2.82 \prec \epsilon_t \prec 3$		3.5
	$3 \prec \epsilon_t \prec 3.78$		4
	$3.78 \prec \epsilon_t \prec 4$		4.5
	$4 \prec \epsilon_t \prec 5$		5

Table 3.3: Monetary Policy when $\sigma = 0.4; \phi = \psi = k = 1; a2 = 0.8$

derived in Section B.1.0.4 in the appendix.

3.4.3 Welfare Comparison between Information Transparency and Information Secrecy

Is information secrecy optimal for the Central Bank in the macroeconomic setting we study? Or, rather, the only justification for the reason a Central Bank might opt for information secrecy lies in the fact that the Central Bank might face some agency problems under information transparency? To the study of this question we turn attention in this section.

We start by fixing ideas and defining the effects of information transparency in our model:

Definition 3.4.2. ***(Information Secrecy and Transparency):*** *The Central Bank is compelled to share with agents its information as to the magnitude of* ϵ_t *under information*

Outcome of the signaling Game: Perfect Separation

Type $\epsilon_\mathbf{t}$	**Strategy** ϵ_j	$(\Delta r)^{a1}$	**Beliefs** $E[\epsilon_t \mid \epsilon_j]$
0	0	0	0
-1,1	-1,1	-0.6,+0.6	-1,1
-2,2	-2,2	-1.17,1.17	-2,2
-3,3	-3,3	-1.74,1.74	-3,3
-4,4	-4,4	-2.30,2.30	-4,4
-5,5	-5,5	-2.86,2.86	-5,5
Off Path	$0 \prec \epsilon_t \prec 1$		2
	$1 \prec \epsilon_t \prec 2$		3
	$2 \prec \epsilon_t \prec 3$		4
	$3 \prec \epsilon_t \prec 4$		5
	$4 \prec \epsilon_t \prec 5$		5

Table 3.4: Monetary Policy when $\sigma = 0.2; \phi = \psi = k = 1; a2 = 0.8$

transparency. This implies that under information transparency agents do not need to condition their expectation on ϵ_t *upon the implemented monetary policy action.*

Instead, the Central Bank enjoys superior and asymmetric information as to the magnitude of ϵ_t *under information secrecy. Therefore, agents need to employ monetary policy to compute* $E\left[\epsilon_t \middle| \epsilon_j\right]$ *under information secrecy.*

It must be remarked that the regime of information secrecy and information transparency assumed in this setting are mere polar cases. In fact, in practice agents might face uncertainty as to how to interpret a given piece of information even under a regime of perfect information transparency. Therefore, even if the Central Bank is fully transparent agents might be reliant on the Central Banker's statements and comments in order to form expectations as to what is the most likely macroeconomic scenario.

We now show that the welfare comparison between information secrecy and information transparency is ambiguous in our setting. However, we can identify some conditions under which information transparency is unambiguously welfare rising and a set of conditions under which, instead, information secrecy is welfare diminishing.

We first identify a scenario in which information secrecy is welfare rising without ambiguity.

Proposition 3.4.4. ***(Secrecy Welfare Rising if a Pooling Equilibrium Obtains):***

It is a sufficient condition for information secrecy to be welfare rising relative to information transparency for a total pooling equilibrium outcome to obtain in the signaling game. This arises when σ *is sufficiently high so that the weight attached to expected capital income in (3.2.27) is sufficiently large.*

Proof. In a total pooling equilibrium under information secrecy each type $\hat{\epsilon}_t$ plays in equilibrium $\epsilon_t = \epsilon_j = E\left[\epsilon_t \middle| \epsilon_j\right] = 0$. Assume that the proposition were false. Hence at least one type $\hat{\epsilon}_t$ is better off under information transparency. Hence, for the proposition to be false, at least one type should receive a better payoff by playing $\hat{\epsilon}_j = \hat{\epsilon}_t$ which is associated to beliefs $E\left[\epsilon_t \middle| \epsilon_j\right] = \hat{\epsilon}_t$.

However, if this were true for any type other than $\epsilon_t = 0$, then the pooling equilibrium would unravel as at least one type would have an optimal deviation away from the total pooling strategy. Hence all types for which $\epsilon_t \neq 0$ are better off with the total pooling equilibrium under information secrecy rather than with the perfect separating equilibrium entailed by information transparency.

Note also that type $\epsilon_t = 0$ is clearly indifferent between the two outcomes as they involve the same strategy and the same beliefs.

This proves the proposition.

□

However, welfare secrecy is not welfare rising in all cases. We identify a condition under which information transparency is welfare superior to information secrecy:

Proposition 3.4.5. ***(Information Transparency Welfare Rising when Total Separation with Limit Pricing Occurs):*** *A sufficient condition for information transparency to be welfare superior to information secrecy lies in the signaling game under information secrecy to yield in equilibrium a totally separating outcome where at least one type plays a limit pricing strategy.*

Proof. In a total separating outcome under information secrecy where at least one type plays a limit pricing equilibrium, Proposition (3.4.3) shows the following holds for each possible type $\hat{\epsilon}_t$: either i) $\hat{\epsilon}_t = \epsilon_j = E\left[\epsilon_t \middle| \epsilon_j\right]$; or ii) $E\left[\epsilon_t \middle| \epsilon_j\right] = \hat{\epsilon}_t \neq \epsilon_j$.

If case i) applies to type $\hat{\epsilon}_t$, then this type is indifferent between information secrecy and transparency.

Instead, if case ii) applies, (3.2.28) shows that welfare for type $\hat{\epsilon}_t$ given beliefs $E\left[\epsilon_t \middle| \epsilon_j\right] = \hat{\epsilon}_t$ is minimized by setting $\epsilon_j = \hat{\epsilon}_t$. Hence type $\hat{\epsilon}_t$ is in this case better off with information transparency.

However, under information secrecy with a total separating outcome with limit pricing under case ii) type $\hat{\epsilon}_t$ cannot optimally deviate from the Cho-Kreps equilibrium strategy and set $\epsilon_j = \hat{\epsilon}_t$. In fact, the limit pricing outcome implies that if $\hat{\epsilon}_j = \hat{\epsilon}_t$ agents do not hold the belief that in equilibrium $E\left(\epsilon_t \middle| \hat{\epsilon}_j\right) = \hat{\epsilon}_t$. This is for some other type would also pool to strategy $\hat{\epsilon}_j$. To avoid being pooled to with some other type, type $\hat{\epsilon}_t$ under case ii) must set $\hat{\epsilon}_j \neq \hat{\epsilon}_t$ under secrecy. This proves the proposition. □

An outcome in which total separation with limit pricing applies is illustrated by the simulation outcome of Section 3.4.2.3. We illustrate the intuition for this result by reference to the results of Section 3.4.2.3. Type $\epsilon_t = 4$ is forced to set $\epsilon_j < \epsilon_t$. Hence, this type is forced not to rise rates as much as it would do under information transparency since, if it sets $\epsilon_t = \epsilon_j = 4$, it will get pooled with type $\epsilon_t = 5$ that would rather play $\epsilon_j = 4$ and face beliefs $E\left(\epsilon_t \middle| \hat{\epsilon}_j = 4\right) = 4.5$ rather than play $\epsilon_j = \epsilon_t = 5$ facing beliefs $E\left(\epsilon_t \middle| \hat{\epsilon}_j = 5\right) = 5$.

Type $\epsilon_t = 4$ is afraid of leading agents to believe that a shock larger than what has occurred to their disposable income has taken place leading agents to confuse type $\epsilon_t = 4$ with type $\epsilon_t = 5$ if type $\epsilon_t = 4$ hikes rates as aggressively as it would do as under information transparency; then type $\epsilon_t = 4$ has to play a limit pricing strategy and hike rates by a smaller extent than what would be optimal under information transparency. Note that, however, such limit pricing strategy does not elicit more favorable beliefs under information secrecy that it does under information transparency. Since, under information secrecy, the equilibrium is still one of perfect separation and hence $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t \ \forall \ t$. For this reason, information transparency is welfare rising in this very special case.

Some qualifications to the results of the analysis are in order. First of all, note that the setting we consider in the model is designed in a very specialized manner to study a particular effect, rather than to provide a complete characterizations of the problem facing the policy-maker. Hence, our setting does not take into account the issue of uncertainty. Information secrecy increases uncertainty which might be welfare reducing in that incomplete information does not allow agents to fully incorporate all the available information into their investment and consumption plans.

Secondly, information secrecy might prevent agents from understanding the behavior of the Central Bank. This might, for instance, impair the Central Bank's ability to effect a large movement in the long portion of the yield curve with a small initial movement in its policy instrument as agents cannot understand what is the signaling content of interest rates.

We provide a simple example of how information secrecy can lead agents to allocate resources in an inefficient manner. Assume that a large negative shock to agent's disposable income is forecasted by the Central Bank. However, the Central Bank decides to play a totally pooling strategy so that no information about such shock is conveyed by the Central Bank. Therefore, agents end up not curtailing their spending plans in the present period, which prevents them from carrying out perfect consumption smoothing in the face of a negative income shock. In fact, in future periods agents regret having over-estimated their disposable income before having learnt about the magnitude of the shocks. As a result, agents find themselves to have over-consumed in the initial period and hence they have not, *ex post*, allocated resources efficiently across periods by achieving perfect consumption smoothing.

However, the objective of macroeconomic efficiency is not incorporated in the loss function of (3.2.24). Were the Central Bank to face a penalty from inducing agents to mis-allocate resources across periods by not fully sharing the available information with them, then the result that information secrecy is welfare rising might not apply even in the context of Proposition 3.4.5.

Note that the FED claimed that sharing its information with agents *could have destabilized the markets and induced excessive volatility* (Goodfriend 1986) when it faced a lawsuit in the eighties over its practice of not sharing its macroeconomic forecast with the general public. The results of this section give a formalization to the FED's argument even if agents are not deemed to be irrational: full information induces pro-cyclical wealth effects which, under information secrecy, the FED can prevent if it plays a pooling or a semi-pooling equilibrium so that in some scenarios information secrecy can have a welfare rising effect.

3.4.4 The Effects of Divulging the Forecasts of the Central Bank Through Full Information Transparency or Detailed Minutes of the Meetings

What is the effect of publishing details minutes of the Interest Rate Setting Panel Meetings? Note that the degree upon which the public is informed about the proceedings of the Interest Rate Panel Setting Meeting varies sharply across various Central Banks.

Recall that, as stated in the introductory chapter, procedures adopted by the FOMC provide for the public release of transcripts for an entire year with a five-year lag. Instead, some concise minutes of each meeting are issued a few days after the next regularly scheduled meeting (a lag averaging about six weeks), and a statement pertaining to the Committee's policy decisions is issued shortly after the conclusion of each meeting (Federal Reserve Board 2001).

By contrast, the Bank of England publishes some non-attributed minutes which, though the information cannot be verified, are often claimed to represent a candid account of the actual proceedings. Such minutes are more detailed that the initial minutes published by the FED in that they account for the diverging views arising inside the Committee.

In sharp contrast with the procedure adopted by the Bank of Engalnd, the ECB plans to publish its minutes with a lag of seventeen years (Buiter 1999). It might therefore be interesting to wonder whether the model of this chapter yields any insight as to what is the implication of different institutional arrangements for the publication of the Interest Rate Setting Panel's meetings minutes. To study such implications, we need to make some special assumptions in the next definition as to what is the implications of publishing the notes:

Assumption 3.4.1. ***(Effect of Publishing the Notes):*** *The Central Bank is no longer endowed with asymmetric and superior information as to the path of macroeconomic fundamentals whenever it has to publish detailed notes of the Interest Rate Setting Panel promptly after each meeting. Hence, when detailed minutes are published with a very short lag agents know the magnitude of the shocks to their cash flows* ϵ_t *without having to condition their beliefs upon monetary policy.*

Note that, in practice, the publication of the minutes is unlikely to totally remove the asymmetry of information between the Central Bank and the public. This is so for the notes might be incomplete (as it is to some extent the case for the Bank of England's ones) or, even if complete, they might display contradictory views which agents do not know how to appropriately weight. Note that the confidence interval for the macroeconomic forecasts divulged by the Bank of England in the Monthly Inflationary Bulletin is often very wide; therefore, it is not infrequent that, while some members might view the data as indicating an inflationary risk, some other members might decide to put a greater weight on the other tail of the confidence interval. As a result, while some members might argue that the forecasts indicate no indication of excessive weakness in the economy, others might suggest that the some forecast exercise displays an indication that inflation is likely to be well below target in the forecast horizon.

Abstracting from these difficulties and taking Assumption 3.4.1 at face value, we show in the next proposition what is the likely effect on monetary policy of publishing the minutes of the Interest Rate Setting Committee.

Proposition 3.4.6. ***(Effect of Publishing the Minutes):*** *When the minutes of the Interest Rate setting body are published and Assumption 3.4.1 holds, interest rates: i) become less likely to stay on hold; ii) move by a bigger magnitude when they are changed and hence become more volatile.*

Proof. The publication of the minutes entails, given the stated assumptions, that the game becomes one of symmetric information since agents fully know the magnitude of ϵ_t independently of the conduct of monetary policy. Hence the results of Proposition 3.4.1 apply: i) $\epsilon_j = 0$ becomes a more likely outcome relative to information secrecy regime when minutes are published; ii) if $\hat{\epsilon}_t > 0$, $\epsilon_j^{pub}(\hat{\epsilon}_t) > \epsilon_j^{npub}(\hat{\epsilon}_t)$, where the superscript *pub* applies to the optimal strategy when the notes are published, while the superscript *npub* refers to the optimal strategy when the minutes are not published. This, together with (3.2.25), implies that $\left|\Delta r_t^{pub}(\hat{\epsilon}_t)\right| > \left|\Delta r_t^{npub}(\hat{\epsilon}_t)\right|$ when $\hat{\epsilon}_t > 0$; iii) if $\hat{\epsilon}_t < 0$, then Proposition (3.4.1) implies again that $\epsilon_j^{pub}(\hat{\epsilon}_t) < \epsilon_j^{npub}(\hat{\epsilon}_t)$ and hence $\left|\Delta r_t^{pub}(\hat{\epsilon}_t)\right| > \left|\Delta r_t^{npub}(\hat{\epsilon}_t)\right|$ when $\hat{\epsilon}_t < 0$.

$\square$

The intuition behind this result can be illustrated with a simple example. Assume that the Central Bank forecasts that a negative shock to agents' cash flow is likely to occur in the near horizon. As we repeatedly emphasized, it might be tempted to use monetary policy only cautiously under information secrecy in order to prevent large procyclical swings in consumption. This is so for agents, under information secrecy, need to condition their expectations as to magnitude of the shock to their disposable income upon monetary policy.

However, the Central Bank's actions do not risk engendering any deterioration in consumers' confidence when by institutional arrangement the minutes of the Interest Rate Setting Panel's meetings are published. In fact, if the minutes of the Interest Rate

Setting panel are promptly published, the asymmetry in information dissipates; agents know the full magnitude of ϵ_t regardless of the conduct of monetary policy. Therefore, in this case the Central Bank has no incentive to conceal the full magnitude of the shock via a policy of gradualism by playing a pooling or semi-pooling strategy.

Along the lines of this intuitive mechanism, counter-cyclical monetary policy tends to be implemented more often and more aggressively in the context of the model whenever the minutes are published and the information asymmetry dissipates regardless of the conduct of monetary policy.

However, the results of this section need to be very strongly qualified. We would like to develop some qualifications on the fact that the publication of the minutes in itself means that monetary policy has no incentive to affect agents' expectations about the underlying dynamic of macroeconomic fundamentals. It must be borne in mind that the minutes, riddled with often contradictory arguments and uncertain predictions, need to be interpreted. Hence, the monetary policy decision might be the clearest signal of the Central Bank's interpretation of the information discussed in the minutes.

3.4.5 The Effects of Altering the Parameters of the Model

What is the implication for the results of the model of altering the responsiveness of investment to monetary policy which is governed by parameter ϕ in (3.2.21)? And what is the implication of increasing the loss associated to deviations of inflation from its zero target which is captured by the parameter ψ in (3.2.27)? We address these questions in this section in which we summarize some further results from the simulation exercise.

We start the analysis of this section by carrying out a simulation exercise whose results is reported in Table 3.5. Recall the weight on the $E\left[\epsilon_t \middle| \epsilon_j\right]$ term in equation (3.2.21) is increasing in σ; hence, the impact of agents's expectations as to the magnitude of the disposable cash flows shock ϵ_t upon aggregate demand is also increasing in σ. Therefore, as confirmed by the results of Section 3.4.2, there is always a threshold value for σ below (above) which the signaling game yields a total pooling (separating) equilibrium. In fact, if σ is sufficiently large (small), the pooling effect wins over (is dominated by) the separating effect and the a pure pooling (separating) equilibrium obtains. This observation must be borne in mind to understand the results of Table 3.5.

Other Parameters Fixed at
ψ*=k=1, a2=0.8*

ϕ	$\sigma <$ **Threshold for Separation no Limit Pricing**	$\sigma >$ **Threshold for Total Pooling**
0.5	0.08	0.16
0.6	0.11	0.22
0.7	0.15	0.30
0.8	0.19	0.38
0.9	0.23	0.47
1.0	0.28	0.57
1.1	0.33	0.67
1.2	0.38	0.77
1.3	0.44	0.88
1.4	0.49	0.99
1.5	0.55	1.10
1.6	0.61	1.22
1.7	0.67	1.34
1.8	0.73	1.46
1.9	0.79	1.58
2.0	0.85	1.70

Table 3.5: Simulation Results from varying ϕ

Other Parameters Fixed at
ϕ=k=1, a2=0.8

ψ	$\sigma <$ **Threshold for Separation no Limit Pricing**	$\sigma >$ **Threshold for Total Pooling**
0.5	0.50	1.01
0.6	0.43	0.87
0.7	0.38	0.77
0.8	0.34	0.68
0.9	0.31	0.62
1.0	0.28	0.57
1.1	0.26	0.52
1.2	0.24	0.48
1.3	0.22	0.45
1.4	0.21	0.42
1.5	0.19	0.40
1.6	0.18	0.37
1.7	0.17	0.35
1.8	0.16	0.34
1.9	0.16	0.32
2.0	0.15	0.31

Table 3.6: Simulation Results from varying ψ

The first set of simulations of this section are carried out according to the following procedure. We fix other parameters in the model to take the following values: $\psi = k = 1, a2 = 0.8$. We then let the parameters ϕ across various simulations vary. We aim to calculate the threshold value for σ below which the equilibrium of the game is one of total separation without limit pricing for each examined value of ϕ. We report such threshold value for σ in the first column of Table 3.5.

We then calculate for each value of ϕ in Table 3.5 what is the threshold value for σ above which a total pooling equilibrium applies for the signal game. We report this second threshold value in the second column of the table. We summarize the findings of the simulation exercise in the following proposition.

Proposition 3.4.7. ***(Effect of Increasing Responsiveness of Investment to Monetary Policy):***

When ϕ increases in (3.2.21) and the responsiveness of investment to monetary policy rises, the following obtains under information secrecy: i) the threshold value for σ over which a total pooling equilibrium holds becomes higher and hence a total pooling equilibrium becomes more unlikely; ii) conversely, the threshold value for σ below which a total separating equilibrium holds gets lower so that a totally separating equilibrium without limit pricing becomes more likely.

Proof. The second column of Table 3.5 shows that the threshold level for σ above which a total pooling equilibrium holds is increasing in ϕ. This proves the first part of the statement.

The first column of Table 3.5 shows that the threshold level for σ below which a total separating equilibrium holds is strictly increasing in ϕ. Hence, as ϕ gets higher, a total separating equilibrium is more likely. This proves the second part of the statement.

$\square$

The intuition for this result is analogous to the insight driving Remark 3.3.2. The parameter ϕ governs to what extent a given change in interest rates impacts investment. The higher is ϕ, the greater the incentive for the Central Bank to implement counter-cyclical monetary policy as the separating incentive for the monetary policy game (working through the investment channel of the transmission mechanism) is strong relative to the pooling incentive (which works through the effect of monetary policy on agents' expectations $E\left[\epsilon_t \middle| \epsilon_j\right]$ as to the magnitude of the shock to their disposable income).

We proceed now to study the effect of varying the parameter ψ which is governed by the Central Banks' aversion to inflation. We report in the first column of Table 3.6 how the threshold value for σ below which a separating equilibrium holds is affected by the magnitude of ψ. The second column of Table 3.6, instead, reports how the threshold level for σ above which a pooling equilibrium always obtains varies as we increase ψ. We summarize the results of such simulation exercise in the following preposition:

Proposition 3.4.8. ***(Effect of Varying The Aversion to Inflation):*** *When ψ increases in (3.2.24) so that the Central Banker becomes more averse to inflation, holding other factors constant, the following obtains under information secrecy: i) a total pooling equilibrium becomes more likely since the threshold level for σ above which the perfect pooling equilibrium holds decreases; ii) a total separating equilibrium without limit pricing becomes more unlikely since the threshold value for σ below which a perfect separating equilibrium holds gets lower.*

Proof. As ψ gets larger, the threshold value for σ below which a separating equilibrium holds is shown to diminish in the first column of table (3.5). Conversely, the lower is ψ, the higher must σ be for a totally pooling equilibrium to hold, as shown by the second column of Table 3.5. $\square$

The intuition for this result is as follows. Using, for illustration, expansionary monetary policy not only risks deteriorating consumer's confidence, but also, as shown by (3.2.23), entails inflationary money creation. The more averse is the Central Banker to movements in the price level, the higher is the cost of using active monetary policy. For this reason, the greater is the aversion to inflation, the more is the Central Banker incentivized to play a pooling equilibrium.

Note that the inflation dynamics we have assumed is quite simplistic. In fact, it ignores the effect that the output gap might have upon inflation and it only draws upon monetary elements. However, it might also be realistic to consider that the Central Bank might face uncertainty as to the inflationary impact of its monetary policy action. Hence, upon loosening monetary policy the Central Bank is aware that it might trigger off an acceleration in the rate if inflation. The more averse is the Central Bank to inflation deviating from its target, the more cautious it must be about using monetary policy aggressively.

The results of this section conclude the analysis of the qualitative implications of the model. Before drawing final conclusions, we would like to briefly develop a conjecture as to whether the model can generate a suggestive pattern of interest rate smoothing.

3.4.6 A Conjecture: An Extension To The Model Could Generate a High Continuations to Total Changes Ratio

Can the model generate a pattern of low reversals to total changes ratio? We tackle this question by extending the model slightly. In fact, the model is not designed to study this problem, but we illustrate with a simple example that we can conjecture that an extension of the model could yield an outcome in which there is a slight bias in equilibrium towards continuations relative to reversals under information secrecy; instead, we show that reversals and continuations are equi-probable in the model under information transparency.

It is plausible to assume that, even under *asymmetric information* and informational secrecy, the informational advantage of the Central Bank ought to be short-lived: agents might ignore the magnitude of the shock to their cash flows ϵ_t before the shock occurs, but at time $t+1$ such shock is of full knowledge to them. We therefore assume the following setting:

Assumption 3.4.2. ***(Setting for the Extension):***

The structure of the game we study in the extension is the following. In period t, a set of shocks of magnitude $\epsilon_{j,t}$ occur to agents cash flows in each industry, the magnitude of which is only known by the Central Bank just like in the original game we modeled.

In the following period t+1, no shock (ϵ_{t+1}) occurs but we let $\rho \approx 1$ in (3.2.2) so that $\epsilon_{t+1} \approx \epsilon_t$.

Moreover, we specify both inflation and the money creation equation in terms of levels rather than changes so that (3.2.22) and (3.2.23) are transformed in the extension of the

The Extended Model Under
Information Secrecy

Type $\epsilon_{\mathbf{t}}$	$\epsilon_{j,t}$	r_t	$\epsilon_{j,t+1}$	r_{t+1}
0	0	0	0	0
-1,1	0	0	-1,+1	-0.765,+0.765
-2,2	0	0	-2,+2	-1.46,+1.46
-3,3	0	0	-3,+3	-2.13,+2.13
-4,4	-3.54,3.54	-2.49,+2.49	-4,+4	-2.8,+2.8
-5,5	-5,5	-3.46,3.46	-5,+5	-3.46,+3.46

Table 3.7: Equilibrium Outcome for the Extended Model under Information Secrecy when $\sigma = 0.53; \overline{r} = 0; a1 = \phi = \psi = k = 1; a2 = 0.8$

The Extended Model Under
Information Transparency

Type $\epsilon_{\mathbf{t}}$	$\epsilon_{j,t}$	r_t	$\epsilon_{j,t+1}$	r_{t+1}
0	0	0	0	0
-1,1	-1,+1	-0.765,+0.765	-1,+1	-0.765,+0.765
-2,2	-2,+2	-1.46,+1.46	-2,+2	-1.46,+1.46
-3,3	-3,+3	-2.13,+2.13	-3,+3	-2.13,+2.13
-4,4	-4,+4	-2.8,+2.8	-4,+4	-2.8,+2.8
-5,5	-5,5	-3.46,3.46	-5,+5	-3.46,+3.46

Table 3.8: Equilibrium Outcome for the Extended Model under Information Transparency when $\sigma = 0.53; \overline{r} = 0; a1 = \phi = \psi = k = 1; a2 = 0.8$

model to be:

$$\pi_t = m_t;$$
$$m_t = -\left(r_t - \overline{r}\right)^{a1};\ \overline{r} > 0; \quad a1 < 1; \tag{3.4.3}$$

The transformation of (3.4.3) implies that in the version of the model we employ in the extension inflation depends negatively on the first difference for the level of the real rate and a time-invariant term, rather than on the difference of interest rates as in the original inflation equation (3.2.23); money creation depends also negatively on the difference between the real rate and a given constant.

All other assumptions remain the same as in the baseline model.

We are now position to solve the extended model for the optimal choice of r_t both at time t and at time t+1. Note that, loosely speaking, the rule of thumb to take into account of the transformation of (3.4.3) in the extended model lies in noticing that all the results of the original model hold also in the extension with the slight modification that any term in Δr_t in the original model becomes $r_t - \overline{r}$ in the transformed model.

We follow the following strategy to formulate our conjecture. We carry out two simulations based on the extended model. The first simulation takes place under the assumption of information secrecy, while the second happens under information transparency. Notice that the model under information secrecy at time t resembles very closely our original signaling game. However, at time $t+1$ even under information secrecy the game is one of full and symmetric information since agents know the magnitude of ϵ_{t+1} with full certainty. This prompts the following remark:

Remark 3.4.1. *At time t+1, the game is one of full information and hence the optimal strategy for the Central Bank lies in setting* $\epsilon_{t+1} = \epsilon_{j+1}\ \forall t$ *as shown by equation (3.2.28).*

We now proceed to analyzing the extended game under a certain constellation of parameters under the assumption of information secrecy. We report the resulting equilibrium from this experiment in Table 3.7. We fix parameters to take the following set of values: $\sigma = 0.53; \overline{r} = 0; a1 = \phi = \psi = k = 1; a2 = 0.8$.

The first column of Table 3.7 reports the type ϵ_t that obtains in each case. The second column reports the equilibrium strategy for each type at time t; the fourth column reports the equilibrium strategy for each type at time t+1. The third and fifth column report the equilibrium interest rate at time t and time t+1 respectively.

We now turn attention to the equilibrium obtaining under information symmetry. To summarize the equilibrium under symmetry we carry out an analogous simulation exercise, whose outcome we report in Table 3.8. Each column in this table has the same interpretation as in Table 3.7. Note that in this instance the equilibrium is one of perfect separation (since the Central Bank has no informational advantage over agents) and hence, as shown by (3.2.28), $\epsilon_t = \epsilon_j \forall t$. The simulation exercise indicates an interesting conjecture, which we now formalize:

Conjecture 3.4.9. ***(Information Secrecy Yields a High Continuation to Total Changes Ratio):*** *In the example provided information secrecy biases the continuation to reversals ratio statistic in favor of continuations relative to the information transparency scenario.*

In fact, if we use the equilibrium outcomes for the extended game illustrated in Table 3.7 and Table 3.8 to compute the expected total continuations to total changes ratio, we obtain a statistic of 2:11 under information secrecy and of zero under information transparency.

The intuition for this minor result is as follows. The Central Bank plays $\epsilon_j = 3.54$ if it is, for instance, of type $\epsilon_t = 4$ as can be verified looking at Table 3.7. This is for in the first period asymmetric information implies that the Central Bank must play a pooling strategy with limit pricing as to prevent type $\epsilon_t = 5$ from pooling to type $\epsilon_t = 4$. Hence, the Central Bank changes interest rates only cautiously to ensure that in equilibrium the mild over-heating pattern the Central Bank has observed is not believed by agents to be instead a very large temporary positive innovation to their disposable income.

In the second period, the Central Bank is freed from the problem that its actions might trigger off pro-cyclical wealth effects and hence can tighten again as to ensure that monetary policy is as tight at it is optimal for it to be under information transparency. Hence, in this case the Central Bank ends up carrying out a continuation movement at time t+1.

On the other hand, the Central Bank fully adjusts in a one-off manner at time t interest rates to their optimum level symmetric information optimum when information is transparent. Hence, interest rates are then kept on hold in all cases at time t+1 in the case of symmetric information absent new information at time $t+1$.

This example is only suggestive, but it leads us to conjecture that information secrecy can bias upward the total continuations to total changes ratio in the model. As a result, the assumption that the informational advantage of the Central Bank dissipates over time could cause interest rate smoothing behavior.

3.5 Conclusions and Discussion

The main results of the paper could contribute to the debate on the following five questions: i) Are inertia or gradualism optimal policies so that Central Banks should not be accused of acting *too little too late*?; ii) Why can the Central Bank choose a limit pricing behavior?; iii) Is information secrecy welfare optimal?; iv) What is the effect of forcing by statue Central Banks to publish immediately detailed minutes of the Interest Rate Setting Panel Meetings?; v) Why do interest rates show a high continuations to total changes ratio?

We do not claim that our results provide a definite answer to any of these areas of investigation. We therefore limit themselves to noting that the setting we have analyzed in this chapter has some insights for each of these questions.

Let us adopt the working definition for gradualism as the observation that interest rates do not respond immediately to changes in macroeconomic fundamentals as a given benchmark model would imply. This definition is in line with the discussion in Blinder (Blinder 1997). We find that the *signaling effect* tends to bias downwards the responsiveness of interest rates to a given shock to macroeconomic fundamentals, as we show in Proposition 3.4.1. Under perfect information the Central Bank is tempted to lower aggressively interest rates after observing a recessionary shock. On the other hand, in the context of our model under asymmetric information the Central Bank might be better off by playing a pooling or a semi-pooling equilibrium, moving rates by a small amount and avoiding to trigger off large pro-cyclical wealth effects as agents learn from monetary policy how to assess their future disposable income. We show that the incentive for gradualism is particularly high when a high proportion of agent's disposable income comes from their capital income upon the magnitude of which the Central Bank enjoys asymmetric information in our setting.

We find in this context that limit pricing behavior might apply in a manner analogous to the findings of Milgron and Roberts (Milgrom and Roberts 1982), as shown in Proposition 3.4.3. We could illustrate this result as follows. Assume that the Central Bank had detected a mild recession and hence would lower rates by fifty basis points under perfect information. Under information secrecy, assume that a separating equilibrium is played, so that the Central Bank reveals its type to the public. Would the Central Bank necessarily lower rates by fifty basis points?

The Central Bank might opt in this case to act more cautiously. In fact, if the Central Bank lowers rates by fifty basis points, agents might (rationally) believe that the Central Bank could have observed not only a mild, but actually a rather large shock to output. This is so for also the type that observes a large shock might pool to the fifty basis points loosening move (leaving in this case agents uncertain as to what is the actual macroeconomic outlook for the economy). Hence, the Central Bank acts more cautiously under asymmetric information that it would under symmetric one to prevent agents from believing that the recessionary shock it has observed is large even if a separarating equilibrium is played. This illustrates our finding of limit pricing behavior in Proposition 3.4.3 and would seem to ground in economic theory the excerpts from the Bank of England Interest Rate Committee meeting from the November 1998 meeting.

The investigation of whether information secrecy is welfare optimal carried out in Proposition 3.4.4 and in Proposition 3.4.5 yields ambiguous results. We show in Proposition 3.4.4 that information secrecy is welfare superior when expectations on capital income play a large role in determining consumption behavior and hence the Central Bank plays a total pooling equilibrium under information secrecy. The Central Bank finds it welfare optimal not to be bound to share its private information with the public when agents' expectations drive large pro-cyclical consumption effects. If the animal spirits of the investors are important, the Central Bank favors secrecy. This consideration might be particularly pressing for the FED given that the US enjoys the largest equity market capitalization per capita.

However, information secrecy is not always welfare enhancing in our model. We show this in Proposition 3.4.5 which essentially relies on the fact that a totally separating limit pricing equilibrium is Pareto inefficient for it forces a number of types to costly differentiate themselves from other types. In fact, we show that if, for instance, a mild recession occurs, the Central Bank, while playing a separating strategy, might not be able to lower rates as aggressively as it would under perfect information. The perfect information outcome cannot be implemented under asymmetric information. In fact, some types might find it incentive compatible to deviate from the pure perfect separating equilibrium absent limit pricing behavior . Hence, in this very special case information secrecy can result into a welfare loss.

Whenever the Central Bank is forced to publish detailed minutes of its interest rate setting meeting, Proposition 3.4.6 shows, interest rates are more likely to move and become more volatile. This is for publishing the minutes of the meetings essentially implies that the signaling value of interest rates is diminished. This is so for agents can, in this scenario, elicit the information the Central Bank is endowed with by reading the minutes of the Interest Rate Setting Panel meetings. Hence, in this case the Central Bank does not risk triggering off any wealth effect by implementing a large movement in interest rates so that the pooling incentive dissipates.

Finally, this chapter hypothesizes in Conjecture 3.4.9 that an extension to the model can produce a high continuations to total changes ratio, or at least bias such ratio towards continuations. As the information advantage dissipates over time, a large recessionary shock, for instance, tends to be gradually translated into looser monetary policy under asymmetric information. In fact, the Central Bank plays a semi-pooling equilibrium in the first period when asymmetric information gives it an incentive not to lower interest rates overly aggressively. Information on the shock becomes symmetric in the successive period, so that the Central Bank can finally set the interest rate at the level it would have chosen under information symmetry. In the process, two interest rate changes of the same sign are implemented even though no serially correlated shock has taken place. We hence conjecture that information asymmetries can lead to a high reversals to total changes ratio.

While the results of Romer and Romer (Romer and Romer 2000) indicate that Central Banks enjoy superior information as to the path of macroeconomic fundamentals, we think that the informational gap between the Central Bank and the public might vary at different points in the cycle. The Central Bank might have a specially strong advantage in forecasting turning points, though such hypothesis has never been tested. Were this to be true, then the model developed in this chapter would be particularly relevant at turning points of the economic cycle. For this reason, while we do not believe that the model developed in this chapter might apply very generally, the considerations it suggests might be particularly relevant at turning points of the economic cycle.

Chapter 4

A Learning Model of the Yield Curve and the Partial Adjustment Mechanism for Interest Rates

Abstract

We study a possible interpretation for the observation that short-term interest rates exhibit a partial adjustment mechanism while interest rate changes show a low reversals to total changes ratio. We also investigate whether interest rate smoothing necessarily lessens a Central Bank's capability to quickly react to news about macroeconomic fundamentals.

We construct a learning model of the yield curve whereby agents employ the historical path of short-run rates and the historical correlation of interest rate changes to determine the slope and the steepness of the yield curve. We interpret the credibility of monetary policy as being represented by the Central Bank's capability of affecting a large movement in the medium and long portion of the yield curve with a relatively small change in the current short-run interest rate.

We find that a positive pattern of historical serial correlation in interest rate changes implies that the Central Bank can bring into effect a large movement in the long portion of the yield curve with a small change in short-run rates, suggestive of the fact that a low reversals to changes ratio and partial adjustment behavior do not necessarily imply an excessively timid response to macroeconomic shocks.

We justify the assumption that short-term rates enter the Central Bank's quadratic loss function together with the rate of expected inflation, and we show that this makes it welfare rising for the Central Bank to be able to affect a large change in long-rates with only a small change in short-run rates. We show that the short-term rate is increasing in its lag and in its lagged rate of change so that monetary policy exhibits a partial adjustment mechanism. We also find that the short-term rate shows a short-run path dependent behavior.

KEYWORDS: Yield Curve Modeling, Partial Adjustment Mechanism for Interest Rates.

4.1 Introduction

It can be recalled from the remarks developed in the introductory chapter that Central Banks are often accused of adjusting monetary policy *too little and too late* in response to forecasted macroeconomic shocks, a claim stemming from two observations: (i) Central Banks smooth interest rate changes so that interest rates follow a partial adjustment mechanism; ii) and that, in the words of Goodhart ((Goodhart 1997),p.1): *" instead of adjusting interest rates by a large enough jump whenever inflation begins to deviate from its desired path, the authorities prefer to make relatively small changes... the consequence is therefore a series of relatively small interest rates changes in the same direction"*.

These two observations have sparked a heated debate as to whether Central Banks are excessively inertial in the implementation of monetary policy (see *inter alia* Goodhart (Goodhart 1997), Ball (Ball 1999) and Rudebusch (Rudebusch 1998)). The subject of the debate could perhaps be summarized as revolving around the following question: Does the smoothness in the short-run rate really imply that the Central Bank's response to a shock is a timid one?

Our analysis stars by recognizing that the short-run rate is not the main indicator of the monetary policy stance. In fact, investment decisions are based on the medium and long portion of the yield curve (Goodfriend 1991). Therefore, the main function of the short-term interest rate lies in affecting the medium and long part of the yield curve through the signaling value of short-term rates. If the Central Bank manages to bring about a large movement in the long portion of the yield curve with only a small movement in the short-term rate, it can still lean aggressively against the wind of macroeconomic shocks even if interest rates adjust by small steps, rather than by rapid and large movements.

Is the steepness of the yield curve endogenous to the conduct of monetary policy? We answer this question in the affirmative in this chapter by constructing a yield curve model in which agents employ forward rates to determine long-term rates via a term structure theory of the yield curve. We also model a mechanism by which agents learn gradually from the past conduct of monetary policy how to set expectations for forward rates.

And why do interest rates exhibit partial adjustment and short-run hystherysis? We propose an explanation for such pattern of behavior based upon the Central Bank's effort to preserve the signaling value of the short-term rate, which, we show, is crucial in ensuring the effectiveness of monetary policy.

Before proceeding further, it might be useful to summarize the stylized facts which motivate the interest rate smoothing literature, to which we refer as a useful benchmark throughout the chapter.

The empirical literature maintains that interest rates follow a partial adjustment mechanism (see, *inter alia*, Clarida et al. (Clarida, Gali, and Gertler 1999), Woodford (Woodford 1999) and Sack et al. (Sack and Wieland 2000)). This is tested by fitting the following expression and checking whether it can be maintained that the lagged level for the nominal interest rate does not determine the current rate. If the null hypothesis that $\rho = 0$ cannot be rejected, empirical testing implies that no partial adjustment mechanism applies:

$$i_t = \rho i_{t-1} + (1-\rho)\left[(rr^* + \pi_t) + \alpha(\pi_t - \pi^*) + \beta y_{t-1}\right]; \tag{4.1.1}$$

This specification states that the current nominal short-term rate is determined by the lagged one month short-term rate, the exogenously determined equilibrium interest rate rr^*, the deviation of inflation π_t from its target π_t^* and the logarithm level of the output gap y_{t-1}. This specification becomes a Taylor rule if $\rho = 0$ so that no partial adjustment applies.

One example of a study of a specification in the vein of (4.1.1) is given by Orphanides and Wieland (Orphanides and V.Wieland 1998), which report the following estimate obtained by instrumental variables for the US economy in the period 1980(Q1)-1996(Q4):

$$\begin{aligned} i_t = \underset{(0.00036)}{-0.0042} \quad & \underset{(0.07)}{+0.795 i_{t-1}} \underset{(0.13)}{+ 0.625\pi_t} \quad \underset{(0.26)}{+1.171 y_t} \underset{(0.23)}{- 0.967 y_{t-1}} + u_t; \end{aligned} \tag{4.1.2}$$

$$\overline{R}^2 = 0.925; SER = 0.010; DW = 2.5;$$

This result indicates that the lagged level of the Fed's Fund target rate is an important determinant of the Fed's Fund Target rate. Clarida et al. (Clarida, Gali, and Gertler

1999) indicate in their survey of the literature that estimates for ρ for the US economy vary across a spectrum ranging from 0.8 to 0.9. Confirming this result, Sacks et al. ((Sack and Wieland 2000),p.208) report in their survey of the interest rate smoothing literature that the finding of partial adjustment in the setting of the short-term interest rate is: "greater than what can be attributed to the systematic policy responses to persistence in output and inflation fluctuations.. and is robust to other specifications, such as rules that respond to forecasts".

We recall a second important source of evidence for the existence of partial adjustment behavior. Goodhart (Goodhart 1997) constructs an interesting statistic to capture the pervasiveness of the observation that interest rate changes are positively serially correlated by constructing a ratio between the number of reversals and the number of total changes which we have slightly updated in Table 1.1 presented in the introductory chapter.

Table 1.1 shows that the reversals to total changes ratio for non-market based short-term rates (the typical instrument of monetary policy) typically range between 1:4 to 1:9. Note, for instance, that the Bank of England, as of October 2001 and ever since it was granted independence, has carried out only three reversals out of twenty-three total changes. Similarly, the ECB has carried out a singe reversal in May 2001, which implies that it enjoys a ratio of reversals to total changes of one to nine.

It might be tempting to conclude that the ECB is more averse to reversing the direction of interest rate changes that the Bundesbank, though a comparison between the rate of reversal of the ECB and the one associated to the Bundesbank is probably devoid of significance given that the sample is short and that reversals are rare events, so that a single additional reversal for the ECB can pivot significantly the results of the comparison.

However, the implication of Table 1.1 seem at the qualitative level robust across countries. We can view this, following the observations of Goodhart ((Goodhart 1997), p.124), as a second source of evidence that Central Banks smooth interest rate changes, follow a partial adjustment rule and that they are reluctant to invert the direction of interest rate changes.

It might be useful at this stage to review the main possible suggestive explanations for interest rate smoothing behavior found in the literature, which we can divide into three families of models: i) accounts for interest rate smoothing and partial adjustment based on model uncertainty; ii) models based on data uncertainty; iii) models based on forward-looking behavior, to which this chapter belongs.

The first family of models to account for partial adjustment and low reversals to changes ratios, including the important contributions of Brainard (Brainard 1967) and Wieland (Wieland 1998), starts off observing that the policy-maker enjoys only a partial knowledge of the magnitude of the parameters which govern the underlying model of the economy. This family of models usually assumes that the policy-maker does not know the slope of the Phillips curve possibly because the parameter specification of the Phillips curve is not time-invariant.

Why would policy-makers in this setting react to a large shock to, for illustration, inflation with only a timid increase in interest rates? This is so for a large movement in the monetary policy instrument is associated excessive uncertainty so that the Central Banker might prefer to enact a small movement and wait until the results of this first experiment are obtained before proceeding to a further hike. The second innovation in monetary policy would then take place when the Central Bank has a better understanding of the true magnitude of the slope of the Phillips curve.

This mechanism can account both for the partial adjustment mechanism for interest rates and for a low reversals to total changes ratio. However, a number of qualifications are in order.

First of all, no interest rate smoothing behavior is observed when the uncertainty is of an additive nature. Only a specification that links the uncertainty resulting from a policy move in a multiplicative way to the policy instrument can generate interest rate smoothing behavior. This is so for additive uncertainty implies that the magnitude of the innovation in the level of interest rates is independent of the amount of uncertainty with which the Central Banker can assess the impact of monetary policy. Instead, under multiplicative uncertainty the larger is the change in interest rates the more the Central Bank shall be uncertain about the outcomes of monetary policy in terms of output and inflation stabilization.

Therefore, unless multiplicative uncertainty is present, model uncertainty cannot explain interest rate smoothing.

A second problematic aspect of this class of models lies in the fact that the results hinge crucially, as in other areas of economics, on the sign of the third derivative of the loss function with respect to a deviation of a given variable from its target level. If, for illustration, the Central Bank targets inflation and the third derivative of the loss function with respect to inflation is positive, the model would imply under-testing and the cautious behavior described above is optimal. However, if the third derivative of the loss function is negative, the model implies over-testing and policy-makers react to a shock more aggressively than they would under the no-uncertainty benchmark.

A second class of models which might be relevant to this problem (see, for instance, Orphanides et al. (Orphanides and Wieland 1998) and Smets (Smets 1991)) studies the implications of data uncertainty, a very central problem to monetary policy as policy-makers observations of macroeconomic variables are likely to be marred by measurement errors.

This class of models is quite successful in explaining why the response of monetary policy to news on macroeconomic variables is more timid than what would be optimal in a model without data uncertainty. The intuition for this result can be gauged with a simple example. Assume that the policy-maker observes a steep rise in forecasted inflation. However, the policy-maker is aware that such unusual value for inflation might be due to a measurement error, and hence uses an adjustment factor to control for the likely over-statement of inflation. Hence, in spite of the sharp rise in measured inflation, the policy-maker reacts to news with only a timid policy response.

It is often noted (see for instance (Sack and Wieland 2000), p.218) that it has not been proved to date, however, that this kind of models can even theoretically account for partial adjustment behavior. This is for this class of models exhibits the certainty-equivalence property *after* that the Central Bank adjusts for the measurement error as to obtain an unbiased estimate for the variables relevant for monetary policy. The process of filtering out the measurement error implies that the certainty-equivalence measure the Central Bank uses to set monetary policy is less volatile than the actual path of the relevant macroeconomic fundamentals, which leads to a smoother path for monetary policy than what would be implied if the Central Bank disregarded the measurement error problem. In this sense, we often refer to this class of models as delivering interest rate smoothing results.

However, once the Central Bank has derived its estimate of the real path for the relevant variables, monetary policy is set as it would be under the no-uncertainty benchmark. For this reason this class of models has not delivered so far results by which interest rates follow a partial adjustment mechanism.

A third and very recently developed area of the literature, to which this chapter belongs, focuses on the forward looking aspect of agents' expectations. Important contributors include Woodford (Woodford 1999) and Levin et al. (Levin, Wieland, and J.Williams 1999) but our results have been independently derived. This family of models has also been somewhat anticipated by an observation by Goodhart (Goodhart 1997) which stressed, without providing a formal model, that a Central Bank which smoothes short-term rates might still implement its inflation targeting mission effectively as long as the long portion of the yield curve is sufficiently reactive to changes in the short-term rate. This intuition permeates all the papers in this area of the literature.

It must be stressed that our results hold under discretion, whereas the results of Woodford and Levin at al. hold under a regime in which the Central Bank operates under commitment. Whereas under commitment the Central Bank is bound to change interest rates according to a given rule it sets advance, in a discretionary model such as ours, the Central Bank is free to re-optimize its choice for the rules followed by monetary policy at all stages.

We could at this stage pre-view the main intuitions behind our model. We first notice that the relevant indicator for monetary policy lies in the medium and long portion of the yield curve. This is for for borrowing for investment purposes usually requires medium or long maturities, rather than short ones. We, therefore, notice that the main function of short-rates is to carry out a *signaling task*, whereby agents employ current short-rates observations to form expectations as to determine forward rates. Then, forward rates are employed to determine the medium and long portion of the yield curve via an arbitrage condition usually employed in term structure models of the yield curve.

We then show what strategy the Central Bank needs to follow to ensure that it can drive a large movement in medium and long-run rates with a small movement in short-rates. We show that medium and long-term rates are very responsive to changes in short-term rates whenever the Central Bank is observed to have a proven record for serially correlating interest rate changes and to carry out a low reversals to total changes ratio. In fact, agents attach a very high signaling value to changes in short-term rates whenever they expect a current rise (fall) in the short-term rate to be followed by a wave of further rises (falls).

We then proceed to assume that the Central Bank's loss function is quadratic in inflation and the level for the short-term rate, which we justify in a number of ways. We show that this assumption implies that the Central Bank attaches a positive value to being able to drive long-term rates to any desired value with only a small initial movement in short-term ones. This is so for the Central Bank, to choose a simple illustration, can ensure that the short-term rate is never overly high for a long period of time as long as it is able to effect a large movement in long-term rates with a small change in short-term ones. Were long-term rates quite insensitive to changes in the short-term rate, the Central Bank would be at times forced to effect an immediate and very large hike in short-term rates- which is not optimal since the Central Bank attaches a negative value to high interest rates and the marginal cost of a tightening of short-term monetary conditions is rising in the level of the short-term rate.

Note that assuming that the loss function is quadratic in the short-term real rate does not imply in itself that the Central Bank wants to smooth interest rate changes. This assumption by itself would only imply that the Central Bank, holding inflation constant, would like interest rates to be as close as possible to zero. The important implication of this assumption for our results lies in the fact that a loss function for the Central Bank which is quadratic in inflation and the level of interest rates induces the Central Bank to aim to make long-term rates as sensitive as possible to short-term ones.

We then proceed to show and interpret the result that the model exhibits a pattern of partial adjustment for nominal interest rates and short-run path dependence.

The rest of the paper is in four sections. Section 4.2 constructs a learning model for the yield curve, which is developed to describe the link between forecasted inflation, short-run interest rates and the medium and long portion of the yield curve. We employ this framework in Section 4.3 to study the interest rate setting problem faced by the Central Bank, whose qualitative implications we analyze in Section 4.4. We draw conclusions and highlight some limitations in Section 4.5.

4.2 The Steepness of the Yield Curve and the Credibility of Monetary Policy

While monetary policy operates directly by affecting an important benchmark measure for the *short-run nominal interest rate*, agents are likely to base aggregate demand decisions on medium and long-run expected real interest rates, as noted by Walsh ((Walsh 1998), p.448).

It is therefore crucial to understand how a change in the current short-term nominal rate affects the medium and long portion of the yield curve. In fact, monetary policy is not likely to be successful in affecting consumption and investment decisions if, while modifying the short end of the yield curve, it has a minimal effect on the medium and long-run interest rates.

On the other hand, the effect of even a small innovation in monetary policy is especially magnified if lowering (rising) the short end yield, lowers (rises) the long-term yield by a great factor.

Recent events are quite illustrative of how important is the link between short-term and long-term interest rates. For illustration, on the 23th of August 2001, while the FED's fund target rate and the yield on the two years bond stood at 350 basis points and 373 basis points respectively, the 30 years bond traded at a relatively high yield of 564 base points. Such failure of long-run rates to respond to the easing in monetary policy was viewed by the Chairman of the FED as a factor dampening the effectiveness of monetary policy, as hinted in one of his testimonies to Congress (Greenspan 2001).

However, in this specific instance, the failure of long-term rates to respond to changes in short-term rates was attributed to factors outside the control of the Central Bank, such as the projected loosening of the fiscal stance- triggering off the expectation of a future increase in the supply of government bonds and hence a fall in their price.

The aim of this section is, given the importance of long-term rates outlined above, to model how the short-term interest rate affects the long-term rate and, only through this channel, it then impacts upon aggregate demand and inflation.

We proceed in three steps. We first outline in Section 4.2.1 how the current interest rates feeds upon the long-run interest rate. To accomplish this, we first study in Section 4.2.1.1 how agents determine forward interest rates by taking into account the information content (that is, the *signaling value*) of the current short-term interest rate. We then investigate in Section 4.2.1.2 the process by which the forward rates determine long-term rates. We do so by employing a simple term structure model of interest rates.

We then study in Section 4.2.2 how agents learn from Monetary Policy how informative the current interest rate is in determining the future forward rate. This section, therefore, studies how the conduct of monetary policy affects the link between short-term and long-term rates in our model. Finally, in Section 4.2.3 we investigate how a measure of the long-run interest rate impacts upon aggregate demand and inflation.

4.2.1 A Simple Operational Model of the Term Structure of Interest Rates

We now employ a term structure theory of interest rates to build a model of the long-run real and nominal interest rate. The term structure theory of interest rates, as developed, for instance, by Cox and Ingersol (Cox and E.Ingersol 1985) and Dahlquist and Svensson (Dahlquist and L.Svensson 1996), implies that the long-run nominal interest rate is determined by an arbitrage condition with respect to forward rates, to which we now turn attention.

4.2.1.1 Determining Forward Rates

We do not assume that agents make explicit use of the Central Bank's model to determine forward rates. Instead, we assume that agents learn continuously from past realizations of monetary policy and adjust the model they employ to determine forward rates at each period. We show in Section 4.4.3 that the model employed by agents is at the qualitative level consistent with the behavior of the Central Bank, though it must be stressed that it is not a rational expectations model and hence it can make systematic forecasting mistakes.

We first define the notation employed throughout the chapter and the assumption about the instrument of monetary policy.

Definition 4.2.1. ***(Instrument of Monetary Policy and Notation):***

The nominal interest rate (expressed in annualized term) occurring between month t+j and month t+j+s is denoted as $i_{t+j,t+j+s}$. *The corresponding (ex-ante expected if* $j > 0$*) real interest rate is denoted with* $E_t\big(r_{t+j,t+j+1}\big)$.

We assume that the only instrument of monetary policy is the one-month nominal interest rate $i_{t+j,t+j+1}$, *which the Central Bank is assumed to fully control without any constraint as long as* $i_{t+j,t+j+1} > 0$.

Agents posit the following *error correction mechanism* to form expectations as to changes in the short-run interest rate, where we define $\Delta i_{t+j,t+j+1} = i_{t+j,t+j+1} - i_{t+j-1,t+j}$:

$$E_t\Big(\Delta i_{t+j,t+j+1}\Big) = E_t\left[\hat{\lambda}_t \; \Delta i_{t+j-1,t+j} + \hat{\mu}_t\Big(\overline{r}_{t+j-1,t+j} - r_{t+j-1,t+j}\Big)\right]; \quad 0 \leq \hat{\lambda}_t < 1, \hat{\mu}_t > 0; \tag{4.2.1}$$

Note that in the expected long-run equilibrium steady state $\overline{r}_{t+j-1,t+j} = r_{t+j-1,t+j}$. Therefore, we can interpret $\overline{r}_{t+j-1,t+j}$ as representing a target rate at which level the real interest rate is expected to settle in the long-run. In fact, the nominal interest rate is expected to be on hold when $\overline{r}_{t+j-1,t+j} = r_{t+j-1,t+j}$.

There are two components to the expected future changes in the nominal interest rate in the right hand side of (4.2.1). The first component captures an expectation that interest rate changes are positively serially correlated. We show in Proposition 4.4.3 that this is consistent with the behavior of the Central Bank in equilibrium.

The second component of the right hand-side of (4.2.1) captures the fact that changes in the nominal interest rate are expected to cease once the real interest rate has achieved a given expected target level.

We can illustrate the qualitative implications of (4.2.1) by employing a concrete example. Table 4.1 records the price of the Fed Funds' Target Rate as of the 29th of January 2001, two days before the meeting scheduled for the FED's FOMC in 2001. Note that the Federal Funds Future contract for a given month is settled in the last day of the

Source: The Chicago Board of Futures and Author's Computations

Settlement Month	**Bid-Ask Price**	**Implied Monthly Av. for FED's Fund Contract**
Jan.	94.015-94.02	5.98
Feb.	94.485-94.49	5.51
Mar.	94.62-94.63	5.37
Apr.	94.84-94.85	5.15
May	94.91-94.92	5.08
Jun.	95.00-95.01	4.99
Jul	95.09-95.10	4.9
Aug.	95.10	4.9

Table 4.1: Fed's Fund Future Contracts Rate as of 29/01/2001

	Meeting Date	**Change**	**Level**
Off-Meeting Move	January 3	-0.5	6.00
	January 31	-0.5	5.5
	March 20	-0.5	5.00
Off Meeting Move	April 18	-0.5	4.5
	May 15	-0.5	4.00
	June 27	-0.25	3.75
	August 21	-0.25	3.5

Table 4.2: The Path of the Target Fed's Fund Rate for the first eight months in 2001

month at a price equal to one-hundred minus the monthly average for the actual FED's fund rate.

We have reported the prices for each traded contract in the table, which we have used to compute a rational-expectations implied estimate for the FED's fund rate average in each month. Note that we have assumed that agents are risk neutral (though it is not *a priori* clear in what directions would risk aversion bias the price of the contracts) and that the FED's fund rate is, on average, equal to the FED's target rate. We also report in Table 4.2 the actual path of the FED's fund target rate as of the 25th of August 2001.

How does the simple model of (4.2.1) qualitatively compare with the expectations we have extrapolated in Table 4.1? First of all, notice that the path of expected Fed Funds rates does indeed display positively serially correlated changes. In fact, as of the 29th of January agents expected a full 50 basis points cut at the next 31st of January meeting following the previous cut on the 3rd of January. We can deduce this by the fact that the February contract priced in a 5.51 basis points average FED rate. Furthermore, agents were pricing in one more cut by the beginning of March and attached a high probability of a further cut to occur in May. A small probability for a cut in interest rates was also priced in for the June contract.

It turns out that agents seem to have underestimated the frequency and magnitude of FED's easing, as shown by Table 4.2. In fact, the FED cut rates at all of the FOMC's meetings scheduled in the time horizon under consideration, and, on top of that, also cut interest rates in the course of two off meeting decisions. However, both the implied expectations as of the 29th of January and the actual path of interest rate changes show a marked pattern of serial correlation, consistently with agents's simple adaptive predictive rule assumed in (4.2.1).

We can also observe from Table 4.1 that agents expected the FED's to converge to 490 basis points by July through a wave of serially correlated and gradually smaller adjustments. Therefore, in this example we could visualize the expected steady state rate $E_t\left(\overline{r}_{t+j-1,t+j}\right)$ to be about 490 basis points.

We assume agents to employ (4.2.1) to compute forward interest rates, which we now define:

Definition 4.2.2. ***(Forward Interest Rates):***

We denote with $i^{f,t}_{t+j,t+j+s}$ the interest rate forward contracted at time t for the rate of interest to be paid between time $t+j$ and $t+j+s$. The forward rate of interest is agreed upon by two contracting parties fixing on a risk-free rate of interest to be applied between time $t+j$ and $t+j+s$.

Forward rates shall be equal to expected rates under some particular conditions. This occurs if there exists at least one risk-neutral agent willing to enter in all forward rate transactions. Under this very special case, which we adopt as a useful and simplifying benchmark:

$$i^{f,t}_{t+j,t+j+s} = E_t\left(i^{f,t}_{t+j,t+j+s}\right); \tag{4.2.2}$$

Computing $E_t(i_{t+1,t+2})$ employing (4.2.1) setting j=1 and s=1 and then exploiting the assumption of (4.2.2) we can determine the forward rate applied between one period ahead and two periods ahead and the one period ahead expected change in nominal interest rates:

$$i^{f,t}_{t+1,t+2} = i_{t,t+1} + \hat{\lambda}_t \Delta i_{t,t+1} + \hat{\mu}\Big(\overline{r}_{t-1,t} - r_{t-1,t}\Big); \tag{4.2.3}$$
$$E_t\left(\Delta i_{t+1,t+2}\right) = \hat{\lambda}_t \Delta i_{t,t+1} + \hat{\mu}_t\Big(\overline{r}_{t-1,t} - r_{t-1,t}\Big);$$

We now compute two periods ahead monthly forward rates. To this end, we first set $j = 2$ and $s = 1$ in (4.2.1) and we substitute (4.2.3) in the resulting expression, and, after applying (4.2.2), we can compute the one period ahead forward rate to be:

$$i^{f,t}_{t+2,t+3} = i_{t,t+1} + \hat{\lambda}_t(1+\hat{\lambda}_t)\Delta i_{t,t+1} + \hat{\mu}_t(1+\hat{\lambda}_t)(\overline{r}_{t-1,t} - r_{t-1,t}) + \hat{\mu}_t(\overline{r}_{t,t+1} - r_{t,t+1}); \tag{4.2.4}$$

Note that the effect of a rise in interest rates at time t on the two periods ahead forward (and on all forwards across the yield curve) is rising in $\hat{\lambda}_t$. In fact, the higher is the coefficient for expected serial interest rate changes, the more agents will revise the future level of the base rate and hence future forward rates after the current base rate is modified.

We could keep proceeding in this fashion and compute a forward rate for all maturities in the yield curve. For the forward rate of maturities in the long portion of the yield curve, the real interest rate should converge to a given target $\overline{r}$, which we do not model explicitly.

In fact, our analytical interest lies in the short-run portion of the yield curve. We make a further simplifying assumption. We assume that, in the short-run, the serial correlation component of interest rate changes is of first order, while the error correction one is of second order. This therefore implies that $\hat{\lambda}_t \gg \hat{\mu}_t$ and therefore that agents use the following adaptive model to determine short-run forward rates:

$$E_t\Big(\Delta i_{t+j,t+j+1}\Big) \approx \hat{\lambda}_t E_t\Bigg[\ \Delta i_{t+j-1,t+j}\Bigg];\ \hat{\lambda}_t < 1; \tag{4.2.5}$$

We can justify the approximation introduced by adopting (4.2.5) relative to (4.2.1) at three levels. Firstly, we are interested in studying how monetary policy affects forward rates at relatively short maturities in the yield curve. In fact, monetary policy can still affect long-run interest rates even by affecting only short-run forward rates, a point we further develop when discussing the term structure theory of interest rates, since the long-run interest rate can be viewed as a basket of one-month forward rates for all months occurring before the maturity of the long-term in question. Hence, if we assume that the serial correlation component dominates in the short-run over the error-correcting one, we can focus the analysis on how the magnitude of the parameter $\hat{\lambda}_t$ drives forward rates.

Secondly, we are interested throughout the paper in studying how $\hat{\lambda}_t$ is affected by monetary policy and if interest rate changes display any serial correlation or whether, on the other hand, agents learn that they should set $\hat{\lambda}_t$ to zero. Therefore, the magnitude of the parameter $\hat{\lambda}_t$ is our primary interest throughout the paper.

Thirdly, agents, assumed here to use an adaptive learning model to determine forward rates, may adopt (4.2.5) as a rule of thumb. In fact, the error correction component of (4.2.1) involves an expected target rate, which agents may not know. Hence agents use only an extrapolative backwards looking mechanism to determine forward rates. If the Central Bank does indeed adjust interest rates to a medium-run target through a series of serially correlated movements, agents may find (4.2.5) a useful rule of thumb to form expectations on future short-run nominal rates and hence to determine forward rates.

Note also that both the nominal and the real interest rate are under (4.2.5) expected to converge to a given bounded value as long as $\hat{\lambda}_t < 1$.

We now turn attention to studying how agents employ (4.2.5) to determine forward rates for any maturity in the yield curve.

Remark 4.2.1. ***(Forward Rates Determination):*** *If agents employ (4.2.5) to determine expectations as to future short-run interest rates, forward rates are linear and increasing in $\Delta i_{t,t+1}$ and take the following form:*

$$i^{f,t}_{t+j,t+j+1} = i_{t,t+1} + \Delta i_{t,t+1} \sum_{s=0}^{s=j} (\hat{\lambda}_t)^s; \tag{4.2.6}$$

Proof. Iterative substitution into (4.2.5) shows that:

$$E_t\Big(\Delta i_{t+j,t+j+1}\Big) = (\hat{\lambda}_t)^j \Delta i_{t,t+1}; \tag{4.2.7}$$

However, assumption (4.2.2), combined with the posited short-run expectations formation model of (4.2.5), implies that:

$$i^{f,t}_{t+j,t+j+1} = i_{t,t+1} + \sum_{s=1}^{s=j} E_t\Big(\Delta i_{t+s,t+s+1}\Big); \tag{4.2.8}$$

Substituting (4.2.7) into (4.2.8) we obtain:

$$i^{f,t}_{t+j,t+j+1} = i_{t,t+1} + \Delta i_{t,t+1} \sum_{s=1}^{s=j} (\hat{\lambda}_t)^s; \tag{4.2.9}$$

This verifies the remark. □

Forward rates are a useful building block since they allow us to compute by arbitrage an interest rate of any maturity in the yield curve, the task we undertake in the next section.

4.2.1.2 Affine Term Structure for Long Term Interest Rates

Are interest rates of all maturities uniquely determined by forward rates? And does the fact that forward rates are linear in $\Delta i_{t,t+1}$ imply that interest rates of all maturities are also linear in $\Delta i_{t,t+1}$? We answer both questions in the affirmative in this section by making use of the term structure theory of interest rates to link forward rates to the shape of the yield curve.

The term structure theory of interest rates derives a long-run interest rate by an arbitrage condition with respect to a set of short-term interest rates. Before proceeding to an illustration of the theory, we define some new notation. Denote with $i_{t,t+m}$ the nominal interest rate which applies at month t to a bond maturing in month t+m. The rate is expressed in monthly terms. Therefore, if m=12 and the yearly rate is, for example, of 1268 basis points, the one year rate expressed in monthly terms $i_{t,t+12}$ is equal to 100 basis points.

For concreteness, assume that an investor is considering the purchase of a bond maturing in m months, which pays a yield equal to $i_{t,t+m}$ in monthly terms. Alternatively, the investor can purchase at time t a set of monthly forward rates and roll over each month her investment obtaining the appropriate forward rate negotiated at time t.

The investor should be indifferent, the term structure theory of interest rates states, between getting the long-term rate (expressed in monthly terms) of $i_{t,t+m}$ which applies to a loan maturing in m months, or rolling forward her investment each month. This condition is verified if, and only if, the following arbitrage relation holds:

$$\left(i_{t,t+m}\right)^m = \prod_{s=0}^{m-1} \left(1 + i^{f,t}_{t+s,t+s+1}\right); \tag{4.2.10}$$

If, instead, the relationship did not hold, there would be an opportunity for an arbitrage (that is, a riskless sure gain) profit to be made. Consider, for instance, a scenario in which the following condition held:

$$\left(i_{t,t+m}\right)^m > \prod_{s=0}^{m-1} \left(1 + i^{f,t}_{t+s,t+s+1}\right);$$

Then any investor could purchase one bond with maturity m and borrow at the one month forward rate. Rolling over the loan each month until period m, the investor would finally receive a monthly yield of $i_{t,t+m}$, which exceeds her borrowing costs. Hence all arbitrageurs will purchase bonds with maturity m until an arbitrage opportunity exists. The arbitrage opportunity shall disappear if, and only if, the monthly yield of the bond with maturity $t+m$ falls by the magnitude required for the equilibrium condition of (4.2.10) to hold again.

Alternatively, we could view this scenario as being one in which no investor wants to hold a bond with maturity *t+m*, but instead all agents prefer to lock in a set of short-run forward rates and keep rolling over their investment until $t+m$. Demand for bonds of maturity *t+m* shall be infinite until the yield falls so that condition (4.2.10) holds.

Conversely, if the following condition is verified:

$$\left(i_{t,t+m}\right)^m < \prod_{s=0}^{m-1} \left(1 + i^{f,t}_{t+s,t+s+1}\right);$$

All investors shall prefer rolling over their fixed income investment until time *t+m* via a set of forward one-month contracts. No agent demands the bond with maturity *t+m* until the price for such bond rises so that condition (4.2.10) is re-established. Until such condition does not hold, arbitrageurs will borrow at rate $i_{t,t+m}$ and invest the proceeds in short-term lending which they roll over through one month forward rates. Such riskless arbitrage strategy drives up $i_{t,t+m}$ until (4.2.10) holds true.

Note that the term structure theory does not hold precisely if transactions costs apply or the markets are not liquid. However, though the term structure theory may not hold precisely, we regard it as a useful theoretical benchmark throughout the chapter. In fact, if the departure from the term structure theory becomes larger than transaction costs, then agents shall find it profitable to exploit the available arbitrage opportunities until the arbitrage condition posited by the term structure theory of interest rates holds.

We now show that the term structure theory of interest rates together with a set of forward rates is sufficient to determine the entire yield curve.

Proposition 4.2.1. ***(Affine Term Structure of Interest Rates):*** *Assume that agents determine forward rates using the model of (4.2.1) and that the term structure of interest rates applies. Hence any interest rate in the yield curve occurring between month t and month t+m is linear in $\Delta i_{t,t+1}$ and equal to:*

$$i_{t,t+m} = i_{t,t+1} + \alpha_m(\hat{\lambda}_t)\Big(i_{t,t+1} - i_{t-1,t}\Big); \tag{4.2.11}$$

Where the term $\alpha_m(\hat{\lambda}_t)$ is increasing in $\hat{\lambda}_t$ as:

$$\alpha_m(\hat{\lambda}_t) = \frac{1}{m}\sum_{s=0}^{m-1}\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j; \tag{4.2.12}$$

Proof. We recall that, if x is small, the following approximation holds:

$$ln\big(1+x\big) \approx x;$$

Taking logarithms from both sides of the term structure arbitrage condition of (4.2.10) and using the above approximation we obtain:

$$i_{t,t+m} \approx \frac{1}{m}\sum_{s=0}^{m-1} i^{f,t}_{t+s,t+s+1}; \tag{4.2.13}$$

Substitute into (4.2.13) the forward rate implied by (4.2.6) to get:

$$i_{t,t+m} = i_{t,t+1} + \frac{1}{m}\sum_{s=0}^{m-1}\left[\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j\big(i_{t,t+1} - i_{t-1,t}\big)\right]; \tag{4.2.14}$$

Letting $\frac{\sum_{s=0}^{m-1}\sum_{s=1}^{j=s}(\hat{\lambda}_t)^j}{m} = \alpha_m(\hat{\lambda}_t)$ in the above expression verifies the proposition. $\square$

The fact that any interest rate in the yield curve is linear in $\Delta i_{t,t+1}$ shall turn out to simplify future computations. This is also a pretty general finding in the fixed income finance literature, which usually, unlike we do, employs models set in continuous time. For instance, Bjork ((Bjork 1998), p.254,256) illustrates a number of short-run forward rate models in continuous time that exhibit an affine term structure for the yield curve.

4.2.2 The Serial Correlation of Interest Rate Changes and the Term Structure

Two important issues on the yield curve model remain to be addressed at this stage. First of all, what is the economic interpretation of $\hat{\lambda}_t$? Secondly, how do agents compute and recalculate at each stage $\hat{\lambda}_t$ in a process of learning from monetary policy? We aim to answer these two questions in the course of this section.

As previously discussed, the effectiveness of monetary policy is enhanced when a small increase (decrease) in the short-run interest rate causes a large increase (decrease) in long-run rates. This happens if interest rate changes are deemed *informative* by agents so that interest rate changes have a great *signaling* value.

A natural measure of how informative interest rates are can, therefore, be gauged by calculating the impact of a change in the current one-period interest rate on medium and long-term rates. This can be computed by differentiating $i_{t,t+m}$ with respect to $i_{t,t+1}$ in (4.2.14):

$$\frac{\Delta i_{t,t+m}}{\Delta i_{t,t+1}} = \left(1 + \frac{1}{m}\sum_{s=0}^{s=m-1}\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j\right); \hat{\lambda}_t < 1; \tag{4.2.15}$$

We therefore introduce and define the concept of the *informativeness of short-run interest rate changes.*

Definition 4.2.3. *We define the informativeness of short-run interest rate changes at time t with respect to the interest rate with maturity m in the yield curve to be:*

$$I_m^t(\hat{\lambda}_t) = \left(1 + \frac{1}{m}\sum_{s=0}^{s=m-1}\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j\right); \tag{4.2.16}$$

Informativeness is therefore increasing in $\hat{\lambda}_t$ and takes on a value of unity when $\hat{\lambda}_t = 0$.

How does the *informativeness of interest rate changes* impact the yield curve? Notice that, in our simple model and as a somewhat overly simplistic feature, the slope of the yield curve is of the same sign as $\Delta i_{t,t+1}$. This can be verified by inspection of equation (4.2.14).

At an intuitive level, this can be explained by noticing that if the Central Bank has increased interest rates in the current period, it has signaled to agents that interest rates shall be hiked in the future as well, which pushes long-term rates above short-term ones and implying a positive slope for the yield curve. Conversely, equation (4.2.14) shows, if interest rates are decreased in the current period agents expect a further easing of monetary policy, which leads them to revise downwards forward rates and hence pushes the long-term portion of the yield curve below the short-term one. In this case the yield curve has a negative slope.

Furthermore, the steepness of the yield curve should be increasing in the magnitude of $\hat{\lambda}_t$. If agents believe interest rate changes to be serially correlated, then a hike in the short-term interest rate at time t should lead to a steepening of the yield curve the more pronounced the greater the magnitude of $\hat{\lambda}_t$.

It, therefore, results that the more interest rates are informative and $\hat{\lambda}_t$ is high, the more long-term rates shall adjust to a change in the short-term rate by a factor greater than a one to one movement, enhancing the effectiveness of monetary policy.

If, instead, agents do not believe interest rate changes to be serially correlated and set $\hat{\lambda}_t = 0$, then a change in interest rates shall just shift the entire yield curve in a parallel way. Conversely, in the paradoxical case that the Central Bank is believed to conduct policy through a number of negatively serially correlated movements, a change in the current base rate may have almost no impact on the entire yield curve.

We can illustrate the importance of the concept of the informativeness of interest rate changes via a simple example. If we let m=*2* in (4.2.15), we can calculate the impact of the current base rate on the yield of a two months bond to be:

$$\frac{\Delta i_{t,t+2}}{\Delta i_{t,t+1}} = \left(1 + \frac{1}{2}(\hat{\lambda}_t)\right);$$

Hence, if the Central Bank changes the one-month interest rate, the short-run portion of the yield curve steeepens if $(\hat{\lambda}_t) > 0$, while it moves in parallel to the change in the one month interest rate if $(\hat{\lambda}_t) = 0$.

A very important caveat is in order. A more realistic model of the yield curve would also incorporate a risk-premium factor, which places a higher yield on forward rates in the long portion of the yield curve, since long-term maturities involve more uncertainty and hence a greater amount of risk than short-run maturities do. For this reason yield curves are usually upwards sloping.

In our model, neglecting a risk-premium factor has the implication that, whenever the Central Bank lowers short-run rates, the yield curve shall be downwards sloping. In practice, yield curves do not always take on an inverted, downwards sloping shape when monetary policy is being eased. For instance, at the time of writing the US yield curve, as previously discussed, is upwards sloping. Therefore, the informativeness of interest rate changes is to be measured by the extent upon which the yield curve flattens, rather than by the extent upon which the yield curve gets inverted as the FED eases monetary policy.

What remains to be determined is how agents shape expectations as to the sign and the magnitude of $\hat{\lambda}_t$. As we are studying a model of adaptive learning, the determination of $\hat{\lambda}_t$ is not implemented by a commitment by the Central Bank. Furthermore, we are assuming that, in the spirit of a learning model, agents gradually adjust $\hat{\lambda}_t$ by using historical observations since they do not know the model of the economy but gradually learn it. Although they are not using the Central Bank's model to determine expectations on future interest rates, the model of (4.2.1) correctly identifies that interest rate changes serially correlate, as shown in Section 4.4.3.

Hence, we assume that agents compute the serial correlation coefficient for interest rate changes by employing an OLS estimate over historical data. This implies that:

$$\hat{\lambda}_t = \frac{\sum_{j=1}^{j=t} \Delta i_{j,j+1} \Delta i_{j-1,j}}{\sum_{j=1}^{j=t} (\Delta i_{j,j+1})^2}; \tag{4.2.17}$$

We make different assumptions as to what the starting point for the sample is in Definition 4.4.1. At this stage, it is maybe easier to think that the sample starts when the Central Banker has taken office, though we propose and analyze different interpretations in Definition 4.4.1.

We have now established how agents use the short-run interest rate to determine all interest rates along the yield curve. We now turn attention to how short-term interest rates feed, via medium and long-term rates, upon inflation.

4.2.3 The Informativeness of Interest Rate Changes and the Impact of the Short-run Interest Rate on Inflation

The first exercise of this section lies in incorporating our previous findings on the behavior of the yield curve into a simple, and not micro-founded, old fashioned Phillips curve model. We also compare the findings from this first exercise with the features of another framework we develop by merging our yield curve model with some features of a fully micro-founded model often employed in the literature (Clarida, Gali, and Gertler 1999).

We start this task by assuming the existence of the following IS relationship, linking the level of output at time $t+q$ to the expected real interest rate accruing between time t and time $t+m$:

$$y_{t+q} = \beta_0 - \beta_1 E_t(r_{t,t+m}) + \zeta_t; q \geq 0; \tag{4.2.18}$$

$$\zeta_t \sim IN(0, \sigma_\zeta^2); \tag{4.2.19}$$

The log of aggregate demand at time $t+q$ is denoted by y_{t+q}, while ζ_t captures a white-noise shock. The real expected interest rate $E_t(r_{t,t+m})$ is, by approximation, equal to the difference between the nominal long-run interest rate $i_{t,t+m}$ and $E_t(\pi_{t,t+m})$, the expected rate of inflation between t *and* $t+m$.

Therefore, in our IS curve aggregate demand is determined by a *medium or long-term* expected real interest rate, rather than by the current short-term rate. Unless the short-term rate has a large impact on the medium or long-term interest rate, monetary policy shall not have a large impact on output. However, the term-structure theory of interest rates predicts that short-term rates have in general at least some effects on the medium and long portion of the yield curve.

A more realistic model of the impact of monetary policy on aggregate demand would incorporate interest rates of different maturities in the yield curve since the various channels of the transmission mechanism of monetary policy operate via interest rates of different maturities.

In fact, it is plausible that the investment channel of monetary policy operates via the long-term interest rate since the horizon of business investment is one of several years. However, other channels for the transmission mechanism, such as the credit channel, may operate instead via short-term interest rates. We abstract from all this difficulties by letting demand be a function of the long-term interest rate only.

Moreover, we assume that the effect of the long interest rate on output operates with some lags. Hence, q in (4.2.18) is positive. For instance, the Bank of England ((The Monetary Policy Committee of the Bank of England 1999), p.9) points out that the effect of a change in interest rates on output peaks about twelve months after the change in stance in monetary policy has taken place.

We now assume that the output gap feeds upon inflation via a simple and somewhat old fashioned short-run Phillips curve, which we do not derive from micro-foundations (we shall compare it to a micro-founded version shortly) and simply assume to take the following form:

$$\pi_{t+n+q,t+n+q+1} = \alpha_0 + \alpha_1 \Big(y_{t+q} - y^* \Big) + \epsilon_{t+m}; \quad m > 0; \tag{4.2.20}$$

where:

$$\epsilon_{t+j} = \rho \epsilon_{t+j-1} + \phi_{t+j}; \rho \leq 1; \phi_{t+j} \sim IN(0, \sigma_\epsilon^2); \tag{4.2.21}$$

Recall that $\pi_{t+n+q,t+n+q+1}$ denotes the level for inflation occurring between period t+n+q and period t+n+q+1.

The log of the NAIRU level of output is represented by y^*; inflation is also subject to stochastic shocks, whose structure we assume in (4.2.21). We also assume that whenever the output gap is positive and output is above its nairu level, inflation is expected to accelerate. Conversely, we hold a negative output gap to be deflationary. This assumption is justified by noting that marginal costs are increasing with respect to scale, and hence prices are increasing in the level of aggregate demand.

Micro-founded versions of the Phillips curve, as argued by Roberts (Roberts 1995), would add to the right hand side of (4.2.21) a one period forward expected inflation term and would hold, consistenly with the formulation of (4.2.21), inflation to be increasing in output. Inflation would, in general, be increasing in the one period forward expected inflation rate for some firms have sticky prices and hence need to keep the future price of other firms into account when making their current pricing decision. Our non micro-founded version of the Phillips curve neglects the forward looking inflation component of the Phillips curve so that there is no inflation persistence, which makes our problem more tractable.

The output gap feeds upon inflation with a lag of m periods. To have an idea of the magnitude of such lag, it is worth noting that the Bank of England suggests that the effect of the current level of the output gap on inflation peaks after one year (see (The Monetary Policy Committee of the Bank of England 1999),p.9). Hence, if we set as $n+q$ equal to the time lag after which monetary policy has its maximum effect upon inflation, we could visualize $n+q$ to lie around twenty-four periods- that is, it would take two years for the full impact of the relevant measure of monetary policy to feed fully upon inflation.

Note that monetary policy is usually believed to start its first impact on inflation with a lag of at least six months and therefore equation (4.2.20) also oversimplifies the lag structure with which output and interest rates feed upon inflation. In fact, a more realistic model would let inflation to be a weighted average of a number of lags of the output gap.

We know study the impact of the long-term interest rate on the expected level of inflation by substituting the IS curve of (4.2.18) into the Phillips curve of (4.2.20). This yields:

$$\pi_{t+n+q,t+n+q+1} = \overline{\pi} - \gamma E_t(r_{t,t+m}) + \epsilon_{t+n+q}; \tag{4.2.22}$$

Note that $\overline{\pi} = \alpha_0 - \alpha_1\beta_0 + \alpha_1 y^*$ and $\gamma = \alpha_1\beta_0\beta_1$.

We now incorporate the previously developed yield curve model of equation (4.2.14) into the Phillips curve of (4.2.22). We aim to study how the slope and the steepness of the yield curve affect the impact of monetary policy on inflation. Therefore, we substitute (4.2.14) into (4.2.22) and approximate the expected real interest rate between time t and time t+m $r_{t,t+m}$ as being the difference between the nominal rate $i_{t,t+m}$ for the same maturity and the expected rate of inflation $\pi_{t,t+m}$ from t to t+m to obtain:

$$E_t\Big(\pi_{t+n+q,t+n+q+1}\Big) = \overline{\pi} - \gamma\Bigg[i_{t,t+1} + \alpha_m(\hat{\lambda}_t)\Delta i_{t,t+1} - E_t(\pi_{t,t+m})\Bigg] + E_t(\epsilon_{t+n+q}); \quad (4.2.23)$$

$$\alpha_m(\hat{\lambda}_t) = \frac{1}{m}\sum_{s=0}^{m-1}\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j;$$

$$\epsilon_{t+j} = \rho\epsilon_{t+j-1} + \phi_{t+j}; \rho \leq 1; \phi_{t+j} \sim IN(0,\sigma_\epsilon^2);$$

This relationship highlights how the *informativeness* of interest rate changes affects the Central Bank's capability to control inflation via a small change in short-term rates, as we observe in the next remark.

Remark 4.2.2. ***(Inflation Control and the Signaling Value of Interest Rates):*** *The higher is $\hat{\lambda}_t$ and the more interest rate changes are informative, the more a small change in the short-run interest rate has a large effect on projected inflation*

In fact, (4.2.23) shows that the impact of a change in interest rates on inflation is increasing in $\hat{\lambda}_t$: the larger is $\hat{\lambda}_t$, the more a given change in the short-run rate affects long-run rates and hence the output gap and inflation.

We now briefly compare the short-run relationship between inflation and interest rates here developed with the results obtained by incorporating our yield curve model into a micro-founded IS-LM framework now quite popular in the literature (Mccallum and Nelson 1997).

A Comparison with a Micro-Founded Phillips Curve

The micro-founded model linking inflation to the yield curve here presented belongs to a family of models that assume partial price stickiness, as surveyed in Clarida et al. (Clarida, Gali, and Gertler 1999), and includes, *inter alia*, Kerr et al. (Kerr and R.King 1996) and Nelson et al. (Mccallum and Nelson 1997). We here just reports some results along the lines of Clarida, Gali and Gertler ((Clarida, Gali, and Gertler 1999), sec 2.1).

The output gap, denoted by x_t in logarithmic terms, is described by a micro-founded IS curve which is a linearized first order condition for the choice of consumption:

$$x_t = -\phi\Big[i_{t,t+1} - E_t(\pi_{t+,t+1})\Big] + E_t(x_t) + g_t; \qquad (4.2.24)$$

A white-noise stochastic shock is denoted by g_t, while other pieces of notation are consistent with the previous sections. The output gap is diminishing in the expected level of the short-run interest rate because substitution effects are at work: a high expected real rate of interest renders future consumption cheap relative to current one and hence lowers aggregate demand in the current period. Consumption is also rising in the expected level of future consumption and output as agents try to smooth out consumption across periods.

The authors present the following Phillips curve (which can be derived from micro-foundations), where again u_t denotes a white-noise shock:

$$\pi_{t,t+1} = \chi x_t + \beta E_t(\pi_{t+1,t+2}) + u_t; \tag{4.2.25}$$

This relationship holds as firms attempt to do price-mark up while prices are partially sticky. As previously mentioned, firms have to anticipate the future price level in the current pricing decision since they might not be able to revise prices in all periods. Also, inflation is increasing in the output gap since the marginal cost is assumed to be increasing in the level of output.

We can solve the IS curve for x_t and the Phillips curve for π_t to obtain:

$$\begin{aligned} x_t &= E_t\left[\sum_{j=0}^{\infty} -\phi\Big(i_{t+j,t+j+1} - \pi_{t+j,t+j+1}\Big) + g_{t+j}\right]; \\ \pi_{t,t+1} &= E_t\left[\sum_{i=0}^{\infty} \beta^i\Big(\chi x_{t+i} + u_{t+i}\Big)\right]; \end{aligned}$$

The output gap is diminishing in a weighted average of future expected real rates: the higher the future expected real rate, the more agents substitute expensive present consumption with cheaper future one. Also, solving the Phillips curve for $\pi_{t,t+1}$ shows that the higher are the expected future output gaps, the higher the expected future marginal costs upon which firms have to mark up, hence inflation is increasing in a weighted average of future output gaps.

Finally, substituting the IS curve solved out for x_t into the Phillips curve solved out for $\pi_{t,t+1}$ yields the following expression linking current inflation to future expected real rates:

$$\pi_{t,t+1} = E_t\left\{ \sum_{i=0}^{\infty} \beta^i \left[-\chi \sum_{j=0}^{\infty} \phi \left(i_{t+j+i,t+j+i+1} - \pi_{t+j+i,t+j+i+1} + \frac{g_{t+i+j}}{\chi} \right) + u_{t+i} \right] \right\}; \quad (4.2.26)$$

Note that at this stage no assumption has been made on how agents determine expectations on the future values of the short-run interest rate. For direct comparison of the results of the first exercise linking the rate of projected inflation to monetary policy we can extend (4.2.26) and substitute the assumption we made in (4.2.6) about how agents form expectations on future interest rates into (4.2.26) obtaining:

$$\pi_{t,t+1} = E_t\left\{ \sum_{i=0}^{\infty} \beta^i \left[-\chi \left(\sum_{j=0}^{\infty} \phi \left(i_{t,t+1} + \Delta i_{t,t+1} \sum_{s=i}^{s=j+i} ((\hat{}\lambda)_t)^s \right) - \pi_{t+j+i,t+j+i+1} + \frac{g_{t+i+j}}{\chi} \right) + u_{t+i} \right] \right\}; \quad (4.2.27)$$

Hence equation (4.2.27) super-imposes our yield curve model on a micro-founded model of inflation often found in the literature. We now observe upon points of similarity and differences between the results obtaining inserting our yield structure model in a non-microfounded model as we do in (4.2.23) as opposed to the relationship one gets inserting our yield curve model in a micro-founded framework as in (4.2.27).

The main similarity between (4.2.23) and (4.2.27) is the fact that in both expressions inflation is: linear and negatively related to both $i_{t,t+1}$ and $\Delta i_{t,t+1}$; sensitive to changes in the short-run interest rate by a factor directly proportional to $\hat{\lambda}_t$ and hence also directly proportional to the informativeness of interest rate changes.

We notice, though, that in the micro-founded version inflation depends upon a weighted average of all the expected rates along all maturities of the yield curve. Furthermore, it involves a different discount factor and lag structure than the non-microfounded version of (4.2.23), which we employ throughout the rest of the document for computational simplicity.

We have now fully characterized in this section the link between agents' yield curve model, expected inflation, long-run interest rates (governed by agent's yield model) and short-run rates (controlled by the Central Bank). The background is set for the Central Bank's interest rate setting problem, to which we not turn attention.

4.3 The Central Bank's Problem under Discretion and the Value of Credibility

The final goal of this section lies in deriving the first order conditions for the interest rate setting problem solved by the Central Bank. However, a number of inter-mediate steps are necessary to achieve this objective.

First, we specify in Section 4.3.1 the loss function the Central Bank seeks to minimize.

We then proceed to show in Section 4.3.2 that, under some conditions, the expected loss function the Central Bank faces at time $t+i$ is diminishing, holding other factors constant, in the magnitude of $\hat{\lambda}_{t+i}$.

We then proceed in Section 4.3.3 to finally study the first order conditions for the Central Bank optimal choice of interest rates.

4.3.1 The Objective of Monetary Policy

We assume throughout the rest of the paper that the Central Bank faces a loss function which is quadratic in inflation and the level of the expected short-run interest rate, so that the loss function takes the form:

$$V_t = E_t \sum_{i=0}^{\infty} \beta^i E_t \Big[(\pi_{t+i,t+i+1})^2 + \delta (r_{t+i,t+i+1})^2 \Big]; \beta \leq 1 \qquad (4.3.1)$$

The first argument entering the loss function is the rate of inflation. We have assumed that the inflation target is zero and that welfare loss is symmetric around such target. In practice, the inflation target is positive and a zero inflation target might be undesirable- for quality improvements might actually imply that a zero measured reading for inflation corresponds to an actual fall in the price level once quality improvements are accounted for. Furthermore, if workers are near-rational and suffer from money illusion, real wages tend to experience more downwards stickiness with a zero inflation target than they would have with a positive inflation target. However, assuming a zero inflation target is convenient and without loss of generality for the problem we here analyze.

The assumption that welfare is decreasing in the expected level of the short-term interest rate needs some justification. A first argument for assuming that the Central

Bank's welfare is diminishing in the rate of interest lies in the fact that the Central Bank might wish to minimize the short-term cost of borrowing incurred by consumers. This argument is particularly powerful if mortgages are indexed to the short-term rate rather than to a long-term bond and if home-ownership is widespread. Hence under this criterion the assumption might be more fitting for the economy of the United Kingdom (where most mortgages are indexed to a standard variable rate which is calculated as a mark-up to the base rate) than to the American economy (in which borrowing costs are usually indexed to the medium portion of the yield curve).

Secondly, as noted by Woodford ((Woodford 1999),p16), Friedman (Friedman 1969) argues that the efficient nominal interest rate is slightly negative. Given that the real interest rate is unlikely to be negative (unless in the course of an un-anticipated inflationary shock as the one that has occurred in the 1970's), then under this light the Central Bank should attempt to let short-term interest rate be as low as possible.

Thirdly, Yun (Yun 1996) shows the theoretical possibility that, in the context of a real business cycle model with sticky prices and cash in advance constraints, households do not allocate resources efficiently when choosing between cash and credit goods (storing too much wealth in cash) if the nominal interest rates and real rates are too high. To avoid this from happening the Central Bank might have a preference for low short-term rates.

On a fourth point, it might be conjectured that the Central Bank might draw some popularity from keeping interest rates low, with the short-term interest rate being the most understood measure of interest rates by the public. However, it must be admitted that such political popularity might be of greater benefit to a Government than to an independent Central Bank, whose panel members are supposed to be insulated from political pressure.

Moreover, some members of the interest rate setting body might represent partisan interests that favor a systematically low interest rate. In this context, the short-term interest rate seems the most widely understood and observed measure of how a certain member of the panel is serving the interest of the partisan group that favored her appointment.

As a fifth and final argument, the assumption that the short-term interest rate contributes to the loss function of the Central Bank can be justified if agents are credit constrained and if the appropriate measure to determine the borrowing ceiling is computed as a ratio between the first repayment installment and the household' current income. In such case, the short-term interest rate determines whether the quantity constraint is binding and Central Banks might wish to allow households to implement their borrowing choices in an unconstrained way.

Note that the fact that the short-term rate enters the loss function in a quadratic manner is not, in itself, sufficient to induce the Central Bank to carry out interest rate smoothing. In fact, this assumption merely implies that the Central Bank, absent other considerations, would always try to set short-run real rates to zero. Moreover, this assumption, quite differently to an interest rate smoothing one, would make the Central Bank very aggressive in lowering short-term rates whenever the inflationary assessment allows it to do so.

We show in Section 4.3.2 that this assumption, instead, implies that it is welfare rising for the Central Bank to be able to drive large fluctuations in the medium and long portion of the yield curve with small fluctuations in the short-run rates.

Having justified the specification of our loss function, we now turn attention to studying its properties. For future reference, it is useful to re-write the loss function of (4.3.1) in the following manner:

$$\begin{aligned} V_t &= E_t\Big[\sum_{i=0}^{\infty}\beta^i L_{t+i} + f_t^1\Big];\ \beta \leq 1; \\ L_{t+i} &= \beta^{n+q}\Big(\pi_{t+i+n+q+i,t+i+n+q+1}\Big)^2 + \delta\Big(r_{t+i,t+i+1}\Big)^2; \\ f_t^1 &= \sum_{i=0}^{i=t+n+q-1}\beta^i\Big(\pi_{t+i,t+i+1}\Big)^2; \end{aligned} \tag{4.3.2}$$

Inflation is driven by monetary policy and agents' determination of the yield curve as derived in equation (4.2.23), which we transcribe below for ease of exposition:

$$E_t\Big(\pi_{t+n+q,t+n+q+1}\Big) = \overline{\pi} - \gamma\Bigg[i_{t,t+1} + \alpha_m(\hat{\lambda}_t)\Delta i_{t,t+1} - E_t(\pi_{t,t+m})\Bigg] + E_t(\epsilon_{t+n+q});$$
$$\alpha_m(\hat{\lambda}_t) = \frac{1}{m}\sum_{s=0}^{m-1}\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j;$$
$$\epsilon_{t+j} = \rho\epsilon_{t+j-1} + \phi_{t+j}; \rho \leq 1; \phi_{t+j} \sim IN(0,\sigma_\epsilon^2);$$

This framework, together with the assumption of equation (4.2.17) on how agents determine the magnitude of $\hat{\lambda}_t$, fully specifies the problem faced by the Central Bank.

Note that the Central Bank at time $t{+}i$ cannot affect the projected rate of inflation prior to period $t{+}i{+}q{+}n$. In fact, the current magnitude of the medium or long-run rate $r_{t+i,t+i+m}$ acts on inflation with a lag of $n{+}q$ periods. If, for pure illustration, we let $n+q$ be equal to 24, then monetary policy at time t affects inflation in a horizon of two years of length, but it has no bearing on shorter horizons. Therefore, all terms subsumed in f_t^1 in (4.2.23) are outside the control of the Central Bank at time t.

A first component of the effects of a change in the current nominal interest rate is highlighted by (4.2.23). In fact, the current short-term nominal interest rate affects long-term interest rates and via this channel the projected level of inflation.

However, we now study under which conditions a change in the short-term nominal rate $i_{t+i,t+i+1}$ at time $t{+}i$ also triggers off some second order effects on all other terms L_{t+i+j} of the loss function by affecting the magnitude of λ_{t+i}.

4.3.2 The Welfare Rising Effect of Credibility

We aim in this section to show that the expected welfare for the Central Bank is diminishing in the magnitude of $\hat{\lambda}_{t+i}$. Or, equivalently, we aim to show that the assumption that the Central Bank's quadratic loss function is increasing in the level of short-run rates implies that it is optimal for the Central Bank to set monetary policy in such a way that it ensures that long-run rates are very responsive to short-run ones. Were long-run rates scarcely responsive to monetary policy, the Central Bank might be forced to set at times a very high level for short-run interest rates, which, given the quadratic nature of the loss function, is very costly. Instead, if long-term rates are very responsive to changes in

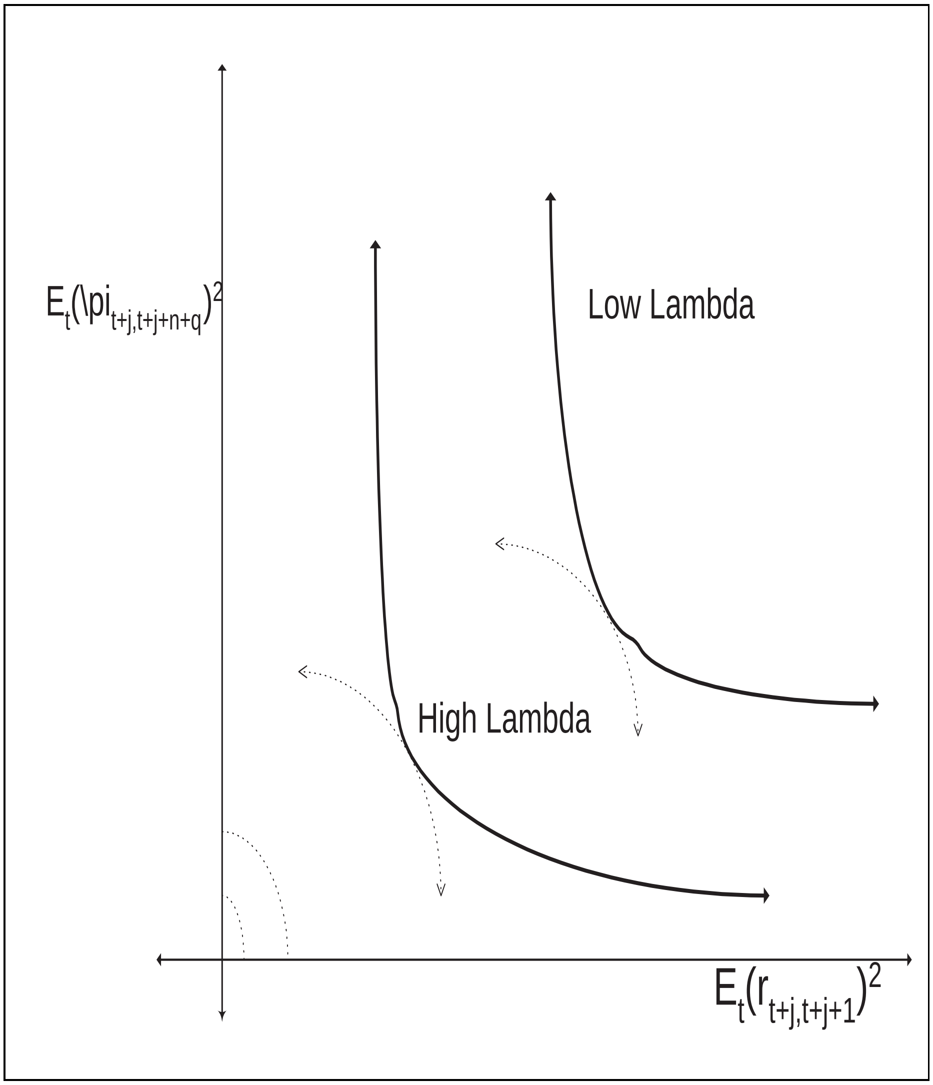

Figure 4.1: Dis-Utility Isoquants and Efficiency Frontiers for $E_t(L_t)$ as a function of $\hat{\lambda}_t$

the short-term rate, the Central Bank can lean against the wind of an inflationary shock by initially having to hike rates by a small amount, which minimizes the average square level for short-term rates. We, therefore, aim to show that:

$$E_t\left(\frac{\partial L_{t+i}}{\partial \alpha_m(\hat{\lambda}_{t+1})}\frac{\partial \alpha_m(\hat{\lambda}_{t+i})}{\partial \hat{\lambda}_{t+i}}\right) < 0; \tag{4.3.3}$$

We illustrate this result before proceeding to deriving it. Figure 4.3.2 depicts the trade-off between the variance of inflation around its target $n+q$ period forward and the square level of the short-term interest rate at time t. These two parameters enter the component of the loss function labeled L_t.

The bliss point the Central Bank would wish to achieve lies where $E_t(\pi_{t+n+q,t+n+q+1})^2 = 0$ and $E(r_{t,t+1})^2 = 0$ since for these combination of values the Central Bank's current component of the loss function L_t achieves its minimum possible value of zero. Therefore we depict a set of concave isoquants in the diagram represents a set of combinations of values for $E_t(\pi_{t+n+q,t+n+q+1})^2$ and $E(r_{t,t+1})^2$ that keep L_t constant.

The diagram also depicts two efficiency frontiers representing, for any given value of $E_t(\pi_{t+n+q,t+n+q+1})^2 = 0$, the lowest possible value of $E(r_{t,t+1})^2$ the Central Bank can achieve *given any value of* λ_t.

For instance, if the Central Bank decides to keep interest rates fixed at all times, then inflation would fluctuate greatly without the wind of monetary policy leaning against the course of inflationary shocks. Alternatively, the more the Central Bank wishes to attempt to lower the fluctuations of inflation around its target, the more the short-run interest rate shall have to fluctuate as monetary policy tightens or gets loosened aggressively to counter deflationary or inflationary shocks.

The two efficiency frontiers depicted in the diagram can be Pareto ranked. In fact, the efficiency frontier the Central Bank faces when $\hat{\lambda}_t$ takes on a relatively large value takes the Central Bank closer to the bliss point than the frontier constraining policy when $\hat{\lambda}_t$ is relatively low. In fact, we show in this section that the efficiency frontier shifts outwards when $\hat{\lambda}_t$ decreases. Hence, $E_t(L_t)$ is decreasing in $\hat{\lambda}_t$.

What is the intuition behind such result? We recall that $\alpha_m(\lambda_t)$, the parameter governing the informativeness of interest rate changes, is rising in the magnitude of $\hat{\lambda}_t$. The higher is $\alpha_m(\hat{\lambda}_t)$, the smaller adjustment in the short-run interest rate the Central Bank has to carry out in order to set the projected rate of inflation in line with its target. This is so for the higher is $\alpha_m(\hat{\lambda}_t)$, the more responsive the expected long-term interest rate is to changes in the short portion of the yield curve and the smaller the term $E(r_{t,t+1})^2$, rising the welfare of the Central Bank.

We proceed to formalise this observation, which we first summarize in the following remark:

Remark 4.3.1. ***(The Value of Informativeness of Interest Rates):*** *An increase* *in the magnitude of* $\hat{\lambda}_t$ *shifts out the frontier of diagram* 4.3.2. *This implies that* $E_t(L_t)$ *is diminishing in* $\hat{\lambda}_t$.

Proof. The idea of this simple proof consists of fixing a given target value for $E_t(\pi_{t+n+q,t+n+q+1})^2$ and then showing that, given any target level of the variance of projected inflation around its target, $E_t(r^2_{t,t+1})$ is increasing in $\hat{\lambda}_t$.

Now let $\Big(E_t(\pi_{t+i+n+q,t+i+n+q+1})\Big)^2 = c_0^2\ \forall i$. To achieve this, the Central Bank employs (4.2.22) and sets the long-run expected interest rate to be:

$$E_t(\overline{r}_{t+i,t+i+m}) = \frac{\overline{\pi} - c_0 + E_t(\epsilon_{t+i+n+q})}{\gamma}; \tag{4.3.4}$$

An overline is applied to r to denote that this is the value of interest rates that achieves $\Big(E_t\big(\pi_{t+i+n+q,t+i+n+q+1}\big)\Big)^2 = c_0^2$.

Notice that:

$$E_t\left[\big(\pi_{t+n+q,t+n+q+1}\big)^2\right] = \big(E_t(\pi_{t+n+q,t+n+q+1})\big)^2 + E_t\left[\Big(\pi_{t+n+q,t+n+q+1} - E_t(\pi_{t+n+q,t+n+q+1})\Big)^2\right]; \tag{4.3.5}$$

Substitute (4.3.4) into (4.2.22) to make clear that the forecast error of inflation depends on the forecast error of the stochastic terms:

$$E_t\left[\Big(\pi_{t+n+q,t+n+q+1} - E_t(\pi_{t+n+q,t+n+q+1})\Big)^2\right] = E_t\Big(E_t(\epsilon_{t+n+q}) - \epsilon_{t+n+q}\Big)^2; \tag{4.3.6}$$

Employing (4.3.6), (4.3.5) and the definition of L_{t+i} given in (4.3.2) and subsuming into k constant terms, we can write the loss function L_t for any given level of c_0^2 the Central Bank chooses:

$$E_t\Big(L_t\Big) = \beta^{n+q} c_0^2 + \delta\Big(\overline{r}_{t,t+1}\Big)^2 + k; \tag{4.3.7}$$

From now on, we formulate the assumption that, while $\rho < 1$ in equation (4.2.23), $\rho \approx 1$ is a very close approximation to ρ as we are dealing with monthly data, so that shocks to inflation can be quite persistent on a month to month basis. For pure illustration, if $\rho = 0.97$, about thirty-one per cent of an initial shock to the rate of inflation decays after one year. Employing (4.2.23) we recall that:

$$r_{t,t+m} = r_{t,t+1} + \alpha_m(\hat{\lambda}_t)\Delta i_{t,t+1} - E_t(\pi_{t+1,t+m}); \tag{4.3.8}$$
$$r_{t-1,t+m-1} = r_{t-1,t} + \alpha_m(\hat{\lambda})_{t-1}\Delta i_{t-1,t} - E_{t-1}(\pi_{t,t+m-1});$$

Employing (4.3.4), assuming $\rho \approx 1$ and denoting with $\overline{r}_{t,t+m}$ the interest rate that ensures that:

$$\big(E_t(\pi_{t+n+q,t+n+q+1})\big)^2 = c_0^2;$$

we can notice that:

$$\overline{r}_{t,t+m} - \overline{r}_{t-1,t+m-1} = E_t\Big(\frac{\epsilon_{t+n+q}}{\gamma}\Big) - E_{t-1}\Big(\frac{\epsilon_{t+n+q-1}}{\gamma}\Big) = \frac{\phi_t}{\gamma}; \gamma > 0; \tag{4.3.9}$$

Substituting (4.3.8) in (4.3.9), letting $\hat{\lambda}_t \approx \hat{\lambda}_{t-1}$ and solving for $\overline{r}_{t,t+1}$ we obtain:

$$\overline{r}_{t,t+1} = \frac{(1+\gamma)\phi_t}{\gamma} + \overline{r}_{t-1,t} - \alpha_m(\hat{\lambda}_t)\Big(\Delta i_{t,t+1-} - \Delta i_{t-1,t}\Big); \tag{4.3.10}$$

Using the definition of the real-expected rate by which $\Delta i_{t,t+1} + \phi_t = \Delta r_t$ and letting $r_{t-1,t} \approx r_{t-2,t-1}$ in (4.3.10), the square of ex-ante expected level of the short-term interest rate turns out to be:

$$E_t\Big(r_{t,t+1}\Big)^2 = E_t\Big(r_{t+i-1,t}\Big)^2 + \frac{(\alpha_m)^2}{(1+\alpha_m)^3}\sigma_\phi^2\left[2 + \frac{(1+\gamma)^2}{\gamma^2}\frac{1}{(\alpha_m)^2}\right]; \tag{4.3.11}$$

The second term is diminishing in α_m. To see that, notice that the derivative of the second term with respect to α_m is negative if, and only if:

$$4\alpha_m - 2\alpha_m^2 - 2(1+\alpha_m)\left(\frac{(1+\gamma)^2}{(\gamma)^2}\right) < 0; \tag{4.3.12}$$

This expression is negative for all non-negative values of gamma, which confirms the statement, since α_m is increasing in $\hat{\lambda}_t$. □

This result establishes that credibility has some positive marginal value. In fact, the more interest rates are informative and the smaller movements in short-run rates are necessary to affect large movements in long-run rates, the higher the expected welfare of the Central Bank. This positive marginal value of credibility is therefore a consideration in the setting of first order conditions for interest rates, to which we now turn attention.

4.3.3 First Order Conditions For Interest Rate Setting

We characterize in this section the first order conditions for the optimal choice of the one month nominal interest rate $i_{t,t+1}$ at time t. The Central Bank seeks to minimize (4.3.2) subject to the projected rate of inflation being driven by the medium portion of the yield curve in the manner described by equation (4.2.23).

We also formulate the simplifying assumption that $n + q > m$. This implies that the lag with which monetary policy feeds through inflation is large enough for a nominal change in interest rates at time t not to affect the projected inflation rate for any of the forward rate maturities that determine $r_{t,t+m}$. This is not entirely realistic, but such assumption simplifies the analysis without loss of generality.

We need to determine at this stage what terms of the loss function the Central Bank impacts at time t when setting $i_{t,t+1}$ both through first and second order effects.

Note that the short-run nominal rate $i_{t,t+1}$ affects both the long-run rate $r_{t,t+m}$ and the term involving the rate $r_{t+1,t+1+m}$, as illustrated by (4.2.23). Observe also that the long-run $r_{t,t+m}$ impacts only upon the projected rate of inflation at time $t+n+q$, as shown by equation (4.2.23).

Therefore, the short-run nominal rate $i_{t+i,t+i+1}$ has a first order effect only upon: i) the projection for inflation at time $t+i+n+q$: $E_t(\pi_{t+i+n+q,t+i+n+q+1})$; ii) the term $\delta(r_{t+i,t+i+1})^2$, capturing the dis-utility the Central Bank draws from high short-run real rates; iii) the projection for inflation at time $t+i+n+q+1$: $E_t(\pi_{t+i+n+q+1,t+i+n+q+2})$.

Beyond these first order effects, the Central Bank triggers off some second order effects when choosing the level of the current short-term interest rate. In fact, the parameter $\hat{\lambda}_{t+i}$, which governs the informativeness of interest rate changes in (4.2.23), is itself a function of the historical serial correlation of interest rate changes in the manner specified by (4.2.17).

Note also that the parameter $\hat{\lambda}_{t+i}$, as Remark 4.3.1 shows, affects $E_t(L_{t+i})$. Hence, the Central Bank must also consider the effect of the current monetary policy action on the level of $\hat{\lambda}_{t+i}$ when setting interest rates. Under this light first order conditions for choosing $i_{t,t+1}$ to minimize (4.3.2) subject to (4.2.23) and (4.2.17) yields:

$$0 = E_t\left\{\frac{\partial L_t}{\partial i_{t,t+1}} + \beta\frac{\partial L_{t+1}}{\partial i_{t,t+1}} + \sum_{j=0}^{\infty}\beta^j \frac{\partial L_{t+j}}{\partial \alpha_m(\hat{\lambda}_{t+j})}\frac{\partial \alpha_m(\hat{\lambda}_{t+j})}{\partial \hat{\lambda}_{t+j}}\frac{\partial \hat{\lambda}_{t+j}}{\partial \hat{\lambda}_t}\frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}}\right\}; \beta < 1 \quad (4.3.13)$$

We now proceed to analyze and write out in detail each term in (4.3.13). To this aim, we substitute (4.3.2), (4.2.23),(4.2.12) and (4.2.17) into (4.3.13), which allows us to write out each term in detail. We start with the first term on the right hand side of (4.3.13):

$$E_t\left[\frac{\partial L_t}{\partial i_{t,t+1}}\right] = 2\delta E_t\Big(i_{t,t+1} - E_t(\pi_{t,t+1})\Big) + \quad (4.3.14)$$

$$+ 2\beta^{n+q}E_t\left[\overline{\pi} - \gamma\Big(i_{t,t+1} + \alpha_m(\hat{\lambda}_t)\Delta i_{t,t+1} - E_t(\pi_{t,t+m})\Big) + E_t(\epsilon_{t+n+q})\right]\Big(-\gamma\big(1 + \alpha_m(\hat{\lambda}_t)\big)\Big);$$

The first term in (4.3.14) captures the dis-utility the Central Bank attaches to the expected deviation of the short-term real interest rate from zero. The second term states that the Central Bank benefits from increasing (lowering) the short-term rate whenever projected inflation is above (below) target. The marginal impact of the short-term interest rate on the long-term rate $r_{t,t+m}$ and hence on projected inflation, equation (4.3.14) shows, is increasing in $(1+\alpha_m(\hat{\lambda}_t))$, the informativeness of interest rate changes.

We now let $i^e_{t+1,t+2}$ be the value for the one period short-term interest rate the Central Bank expects to set at time t. Note that such value does not necessarily correspond to the actual short rate $i_{t+1,t+2}$ the Central Bank chooses at time *t+1*. In fact, the Central Bank might re-optimize at time *t+1* the short-run rate $i^e_{t+1,t+2}$ it had planned at time t to implement at time *t+1*. We now proceed to write out the second term on the right-hand side of (4.3.13) again by substituting for (4.3.2), (4.2.23),(4.2.12) and (4.2.17) into (4.3.13):

$$E_t\left[\frac{\partial L_{t+1}}{\partial i_{t,t+1}}\right] = 2\beta^{n+q+1}E_t\left[\overline{\pi} - \gamma\Big(i^e_{t+1,t+2} + \alpha_m(\hat{\lambda}_{t+1})\Delta i^e_{t+1,t+2} - E_t(\pi_{t+1,t+m+1})\Big) + E_t(\epsilon_{t+n+q+1})\right]$$
$$\Big(\gamma\alpha_m(\hat{\lambda}_{t+1})\Big);$$

This expression shows how the current short-run rate also feeds on the projected of inflation at time $t+n+q+1$ for any fixed value of $i^e_{t,t+1}$: if, for instance, the Central Bank increases (decreases) the interest rate in the current period and then, for illustration, stops changing the short-term rate in the next period, the yield curve would flatten in the next period as agents revise their previous belief that forward rates would change.

Turning attention to the terms involving $\hat{\lambda}_t$, note that (4.2.17) implies that:

$$\frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}} = \frac{\Delta i_{t-1,t}}{\sum_{j=2}^{j=t} (\Delta i_{j,j+1})^2} - \frac{\left(\sum_{j=2}^{j=t} \Delta i_{j,j+1} \Delta i_{j-1,j}\right) 2 \Delta i_{t,t+1}}{\left(\sum_{j=2}^{j=t} (\Delta i_{j,j+1})^2\right)^2}; \tag{4.3.15}$$

Therefore, since in general the term $\left(\left(\sum_{j=2}^{j=t} \Delta i_{j,j+1}\right)^2\right)^{-2}$ is of second order, the following statement holds in general:

$$sign\left(\frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}}\right) = sign\left(\Delta i_{t-1,t}\right); \tag{4.3.16}$$

Therefore, agents revise upwards their estimate of the historical serial correlation of interest rate changes if the Central Bank has just implemented a change in interest rates in the same direction as the one implemented in the last period. Conversely, agents revise downwards their estimate of the historical serial correlation of interest rates if the Central Bank has just inverted the direction of the change in interest rates.

Whenever the Central Bank sets rates in a way that increases $\hat{\lambda}_t$, it induces agents to determine forward rates so that the long end of the yield curve is the more responsive to fluctuations in the short maturities of the yield curve, hence increasing the informativeness of interest rate changes captured by the parameter $\alpha_m(\hat{\lambda}_t)$. Employing remark (4.3.1) and equation (4.2.12) we verify that:

$$sign\left\{E_t\left(\frac{\partial L_t}{\partial \alpha_m(\hat{\lambda}_t)} \frac{\partial \alpha_m(\hat{\lambda}_t)}{\partial \hat{\lambda}_t} \frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}}\right)\right\} = -sign\left\{\Delta i_{t-1,t}\right\}; \tag{4.3.17}$$

The following interpretation can be given to (4.3.17). If the Central Bank keeps implementing interest rate changes of the same sign, interest rates become more informative and hence the Central Bank can affect the long portion of the yield curve even with small changes in the short-run interest rate. This has some positive welfare value since it allows the Central Bank to control inflation even if short-run interest rates exhibit a small variance.

However, the impact on how the Central Bank changes the current interest rate on the parameter $\hat{\lambda}_{t+j}$ stretches beyond the period t. In fact, agents use the information they have learnt at time t at all successive periods in order to determine the historical rate of correlation of interest rate changes. Employing (4.2.17) we observe that:

$$\begin{aligned}\frac{\partial E_t\left(\hat{\lambda}_{t+1}\right)}{\delta i_{t,t+1}} = & \frac{E\left[\Delta i_{t-1,t} + \Delta i^e_{t+1,t+2} - \Delta i_{t,t+1}\right]}{\sum_{j=2}^{j=t+1}(i_{j,j+1})^2} \\ & - E\left[\frac{\left(\sum_{j=2}^{j=t+1}\Delta i_{j,j+1}\Delta i_{j-1,j}\right)2\left(i^e_{t+1,t+2} - i_{t-1,t}\right)}{\left(\sum_{j=2}^{j=t+1}(\Delta i_{j,j+1})^2\right)^2}\right];\end{aligned} \tag{4.3.18}$$

The intuition behind this expression is similar to the one motivating (4.3.17), with the only difference that the Central Bank has also to consider the sign of the term $\Delta i^e_{t+1,t+2} - \Delta i_{t,t+1}$ when considering the expected impact of $i_{t,t+1}$ on $\hat{\lambda}_{t+1}$ for at time *t+1* agents also use the observation of the correlation of interest rate changes at time *t+1* to determine the informativeness of interest rate changes.

If the Central Bank chooses $i_{t,t+1}$ in a way that raises λ_{t+1} , then the Central Bank also increases $E_t(\hat{\lambda}_{t+j})$. With the pure purpose of illustrating this point, notice that, provided n is not too large, we can use the following approximation:

$$\hat{\lambda}_{t+n} \approx \hat{\lambda}_{t+1} + \frac{\sum_{j=t+1}^{j=t+n}\Delta^e_{j,j+1}\Delta i^e_{j-1,j}}{\sum_{j=2}^{j=t+n}(\Delta i_{j,j+1})^2}; \tag{4.3.19}$$

This implies that:

$$sign\left\{\frac{\partial E(\hat{\lambda}_{t+1})}{\partial i_{t,t+1}}\right\} = sign\left\{\frac{\partial E(\hat{\lambda}_{t+2})}{\partial i_{t,t+1}}\right\} = sign\left\{\frac{\partial E(\hat{\lambda}_{t+j})}{\partial i_{t,t+1}}\right\} \forall j \geq 3; \tag{4.3.20}$$

The considerations we have formulated so far allow us to dis-aggregate the interest rate setting problem faced by the Central Bank into two components, to which end we introduce the following definition:

Definition 4.3.1. *We denote with $i^{**}_{t,t+1}$ the choice of optimal interest rate that solves the first order condition of (4.3.13).*

Instead, we denote with $i^*_{t,t+1}$ a useful benchmark, capturing the solution the Central Bank would have implemented if $\hat{\lambda}_{t+j}$ were exogenous instead of being endogenous, so *that:*

$$i^*_{t,t+1} = argmin\ E_t\left(L_t(i_{t,t+1}) + \beta L_{t+1}(i_{t,t+1})\right); \tag{4.3.21}$$

The first component of the interest rate setting problem involves the expectation of the terms L_t *and* L_{t+1}, the only two terms directly affected by $i_{t,t+1}$. We can label this as the *interest rate setting problem if the magnitude of* λ_{t+j} *were exogenous.* If the Central Bank did not have to concern itself with the way agents update their beliefs about the informativeness of interest rate changes, the interest rate setting problem would only consist of controlling these two terms.

However, there is a second component to the interest rate setting problem. This component captures the Central Bank's concern for the magnitude of the parameter $\hat{\lambda}_{t+j}$, which drives the relationship between the short-end and the long-end of the yield curve. This second component, which we label *the reputation component* of interest rate setting or R_t, consists of the following terms:

$$R_t = E_t\left\{\sum_{j=0}^{\infty}\beta^j \frac{\partial L_{t+j}}{\partial \alpha_m(\hat{\lambda}_{t+j})}\frac{\partial \alpha_m(\hat{\lambda}_{t+j})}{\partial \hat{\lambda}_{t+j}}\frac{\partial \hat{\lambda}_{t+j}}{\partial \hat{\lambda}_t}\frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}}\right\}; \tag{4.3.22}$$

This useful decomposition of the considerations affecting the Central Bank when it chooses the current short-term rate. We are now ready to study some interesting qualitative implications of the model.

4.4 Qualitative Behavior of Interest Rates

4.4.1 Triggering Off Large Movements in the Medium-End of the Yield Curve with Small Movements in the Short-End

Can Central Bankers take any policy action to ensure that long-run rates be very sensitive to changes in the short-run interest rate set by the Central Bank? Furthermore, what are the implications, if any, of frequently reversing the direction of interest rate changes as opposed to following the practice of implementing monetary policy through a set of interest rate changes of the same sign?

The framework we have developed can shed light on both of these questions. We remarked in Section 4.2.2 that $\hat{\lambda}_t$, the parameter driving the informativeness of interest rate changes, also drives the relationship between the short end of the yield curve and the longer portion.

The higher is $\hat{\lambda}_t$, the more agents believe interest rate changes to be serially correlated and hence the greater the *signaling value*, that is the impact of the short-run on agents' beliefs about the future path of monetary policy, of a change in the short-run rate. However, short-run interest rate changes shall be the more informative the more they exhibit a pattern of historical serial correlation since agents set the magnitude of $\hat{\lambda}_t$ by learning from the history in office of the Central Banker. This is the idea we articulate in the next proposition.

Proposition 4.4.1. ***(Credibility and the Steepness of the Yield Curve):*** *The higher the magnitude of the coefficient of serial correlation of interest rate changes and hence the higher is $\hat{\lambda}_t$, the steeper is the portion of the yield curve between t and $t+m$. Moreover, the slope of the yield curve is positive (negative) if $\Delta i_{t,t+1} > 0$ ($\Delta i_{t,t+1} < 0$).*

This implies that the higher is $\hat{\lambda}_t$, the more the Central Bank is capable of engendering a large shift in the long-run rate $r_{t,t+m}$ with a small change in the short-run rate $r_{t,t+1}$, with the steepness of the yield curve being equal to:

$$\frac{\Delta r_{t,t+m}}{\Delta r_{t,t+1}} = \left[1 + \frac{1}{m} \sum_{s=0}^{s=m-1} \sum_{j=1}^{j=s} \left(\frac{\sum_{j=2}^{j=t} \Delta i_{j,j+1} \Delta i_{j-1,j}}{\sum_{j=2}^{j=t} (\Delta i_{j,j+1})^2} \right)^j \right] \tag{4.4.1}$$

Hence, if $\hat{\lambda}_t$ is large, the Central Bank can effectively counter a large inflationary (deflationary) shock even if it initially implements a small adjustment to the short-run rate.

Proof. First of all, recall that we have formulated the simplifying assumption that the lag with which monetary policy acts on inflation is such that $i_{t,t+1}$ does not feed back on $E_t(\pi_{t,t+m})$. Hence a change in the nominal interest rate $i_{t,t+m}$ translates into a change in the expected real rate $r_{t,t+m}$ in a one to one ratio. Employing this observation and substituting (4.2.17) into (4.2.15) we obtain:

$$\frac{\Delta r_{t,t+m}}{\Delta r_{t,t+1}} = \frac{\Delta i_{t,t+m}}{\Delta i_{t,t+1}} = \left(1 + \frac{1}{m}\sum_{s=0}^{s=m-1}\sum_{j=1}^{j=s}(\hat{\lambda}_t)^j\right); \tag{4.4.2}$$

$$= \left[1 + \frac{1}{m}\sum_{s=0}^{s=m-1}\sum_{j=1}^{j=s}\left(\frac{\sum_{j=2}^{j=t}\Delta i_{j,j+1}\Delta i_{j-1,j}}{\sum_{j=2}^{j=t}(\Delta i_{j,j+1})^2}\right)^j\right].$$

□

The implication of the proposition is that the steepness of the yield curve is, at least in one respect, endogenous to monetary policy. A Central Banker historically known to carry out a set of serially correlated movements in interest rates shall face a more responsive medium portion of the yield curve than a Central Banker known to reverse the direction of interest rate changes with great frequency. For illustration of this point, note that equation (4.4.1) implies that if short-run interest rates have behaved historically according to a random walk process, then the yield curve shall be completely flat, so that all forward interest rates would in this case be equal to the current short-rate.

Figure 4.2 and Fig 4.3 depict the implications of Proposition 4.4.1. Figure 4.2 shows that agents revise future forward rates upwards by an amount increasing in $\hat{\lambda}_t$ if the Central Bank hikes the current short-term rate. Conversely, the yield curve inverts whenever rates are lowered, displaying a very steep negative slope if $\hat{\lambda}_t$ is large, as shown by Figure 4.3.

In the light of these findings can a Central Bank be necessarily be accused of acting, using a common terminology, *too little too late* whenever it reacts to a projected shock to macroeconomic fundamentals by smoothing interest rate changes? Our model answers this question in the negative. Agents understand the historical pattern of interest rate smoothing. Hence they expect a change in the current short-run rate to have considerable signaling value as to expected magnitude of future interest rate changes, and, in the light of this, small movements in the current base rate are sufficient to trigger off a large change in long-run rates.

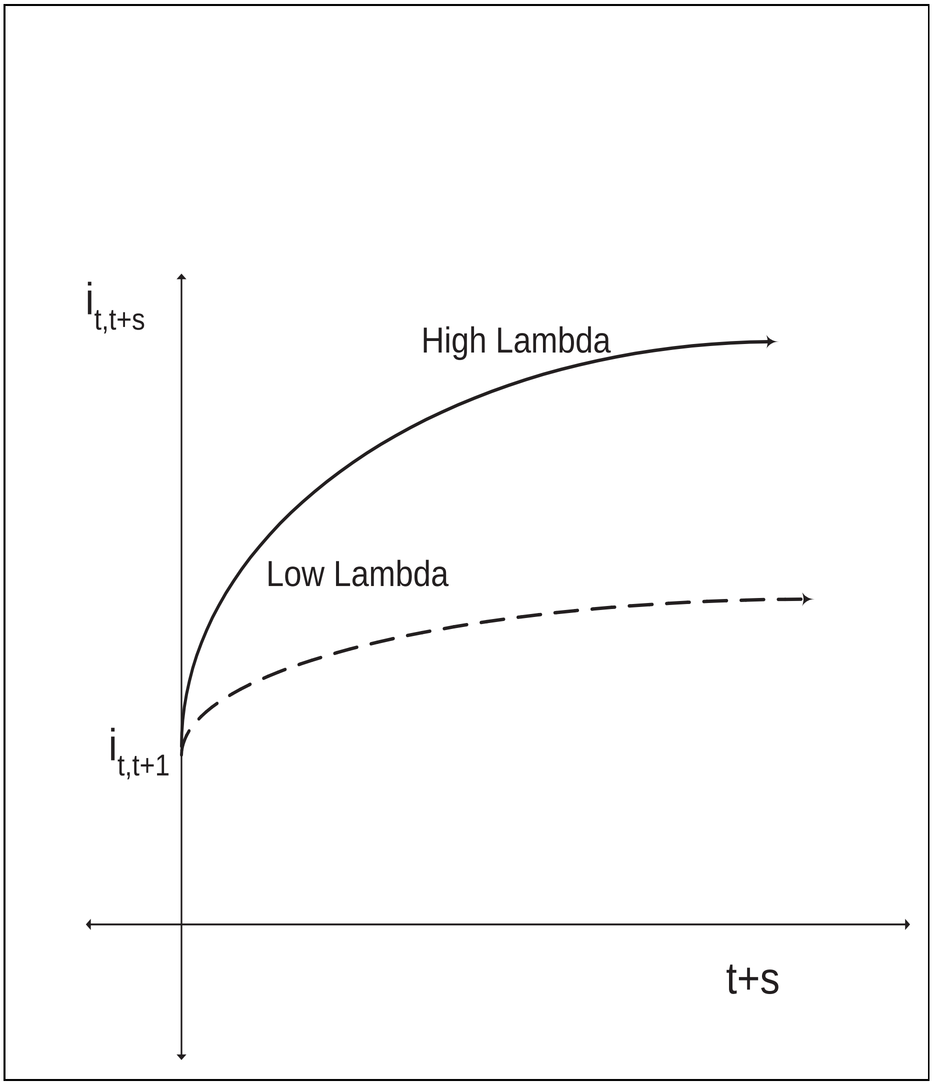

Figure 4.2: The Yield Curve when $\Delta i_{t,t+1} > 0$

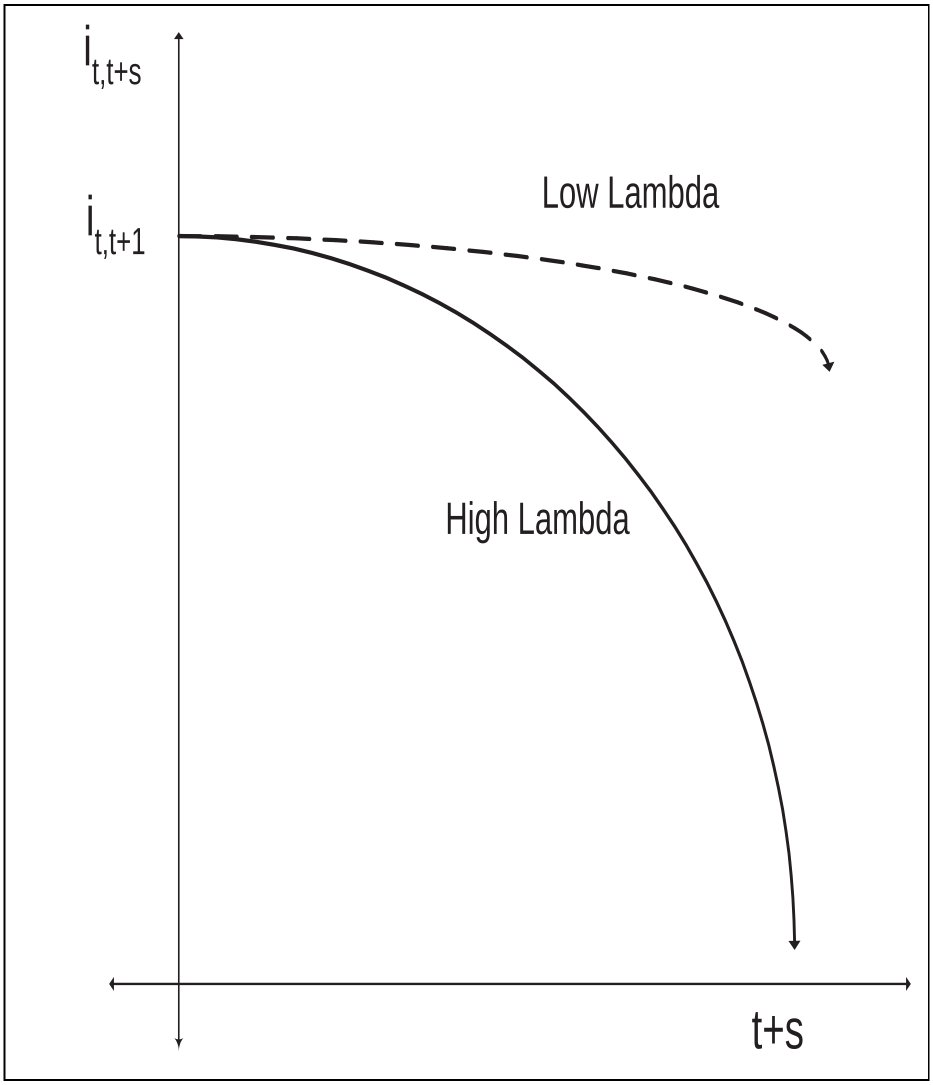

Figure 4.3: The Yield Curve when $\Delta i_{t,t+1} < 0$

We need at this stage to develop a number of caveats. First of all, notice that the yield curve is in reality rarely inverted, since bonds with a long maturity carry a higher yield than bonds maturing in the short-run. The fact that our model cannot characterize an inverted yield curve as a pretty rate event just highlights the implication of omitting risk premia factor. However, we do not aim to fit the yield curve, but just to understand some interesting qualitative properties in the context of the relationship of the yield curve and monetary policy, so that this omission seems to entail no loss of generality for our purposes.

Secondly, one might wonder what forward rate maturities does the Central Bank really affect when changing the current monthly short-run repo rate. It seems unlikely that the Central Bank could have a very large impact upon the very long portion of the yield curve. For illustration, let us consider the yield of the thirty-year bond. This consists of the weighted average of the expectation of the yield on the one month fixed income riskless asset for the next three-hundred and sixty monthly periods. If the current stance of monetary policy is informative for the near horizon (that is to say around the next twenty four months) but not for the long one, it seems almost natural to believe that the thirty-year bond should not fluctuate wildly when the short-run interest rate changes. Instead, the medium portion of the forward yield curve should be highly sensitive to short-run rates since it is driven by forward-rate maturities for the determination of which the current actions of the Central Bank seem to be quite informative. Consistently with such considerations, the two year bond is traditionally held to exhibit the most volatile yield.

Finally, note that the way agents determine $\hat{\lambda}_t$ in our model would be excessively mechanical if our aim consisted of formulating a realistic model of the yield curve. In fact, it might be plausible to believe that agents might condition $\hat{\lambda}_t$ also upon the notion of what is the interest rate level the Central Bank aims to achieve once it has completed its process of adjustment to its target rate. Or, alternatively, agents might believe that the coefficient of expected serial correlation of interest rate changes must be held to be time-varying for various maturities of the yield curve, rather than being held to be uniformly equal to $\hat{\lambda}_t$ throughout the spectrum of forward rates.

However, the aim of our yield curve model is to characterize in a qualitative fashion the relationship between the Central Bank's reputation for following an interest rate smoothing procedure and the relationship between the short-end and the long-end of the yield curve. The extensions to the yield curve model proposed above would not seem to compromise our finding that a small change in the short-run rate has a large impact on the long portion of the yield curve if agents believe that the Central Bank adjusts interest rate through a partial adjustment mechanism and a wave of positively serially correlated changes. This establishes the link between the steepness of the yield curve and the reputation the Central Banker enjoys for smoothing interest rate changes.

This relationship, besides being of interest in itself, plays also a role bringing about a pattern of short-run path dependence in the model, which we study in the next section.

4.4.2 Short-Run Path Dependence of Interest Rates

Are lagged values of nominal interest rates significant in determining the level of the current nominal interest rate so that the model would exhibit some path-dependence property for the optimal interest rate? And if the last question is answered in the affirmative, how many lags enter the determination of $i^{**}_{t,t+1}$? We answer these questions in the following proposition:

Proposition 4.4.2. ***(Short-Run Hysterysis Property):*** *Interest rates are short-run path dependent. Both the level of $i_{t-1,t}$ and that of $i_{t-2,t-1}$ contribute to determine the optimal interest rate $i^{**}_{t,t+1}$ the Central Bank determines at time t.*

Proof. Note that $i_{t-1,t}$ contributes to the component of first order conditions of equation (4.3.14), which measures the marginal contribution of $i_{t,t+1}$ to $E_t(L_t)$. Turning attention to the terms capturing the reputation effects, note also that equations (4.3.16),(4.3.18) and (4.3.19) jointly imply that $E_t\left(\frac{(\partial\hat{\lambda}_{t+j})}{\partial\hat{\lambda}_{t+j}}\frac{\partial\hat{\lambda}_{t+j}}{\partial i_{t,t+1}}\right)$ is a function of $\Delta i_{t-1,t}$. Hence the R_t component of the first order condition defined in (4.3.22) is also a function of $\Delta i_{t-1,t}$. $\square$

We not proceed to illustrate the result of the proposition at an intuitive level. The first factor causing path-dependence in the model can be labeled the *reputation path-dependence effect* and is captured by the terms of (4.3.22). To illustrate this effect, assume that the Central Bank intends to hike interest rates and consider two alternative scenarios. In this first scenario, interest rates have been hiked in the previous period. Therefore, the more aggressively the Central Bank hikes rates in the current period, the greater gain in its reputation for serially correlating interest rate changes it shall reap. Hence, in this first scenario, hiking rates carries at the margin the benefit of rising $\hat{\lambda}_t$ and of inducing, loosely speaking, long-term rates to be in future more responsive to changes in short-term rates.

Consider instead a second scenario in which interest rates have been lowered in the past period and the Central Bank, having observed an acceleration in projected inflation, is considering whether to hike rates or not. Now in this second scenario hiking rates carries at the margin the cost of lessening the Central Bank's reputation for serially correlating interest rate changes. A lower $\hat{\lambda}_t$ entails some welfare cost to be weighted against the benefits of hiking rates aggressively in order to bring inflation in line with the Central Bank's target.

The contrast between these two scenarios highlights at an intuitive level the rationale behind the path-dependency of $i^*_{t,t+1}$ on $\Delta i_{t-1,t}$: the impact of $i_{t,t+1}$ upon $E_t(\hat{\lambda}_{t+j})$ is a function of $\Delta i_{t-1,t}$.

Furthermore, also the *yield curve expectation effect* is at work to bring some path-dependency into the model. Such effect works through the component of (4.3.14) of first order conditions and can be characterized intuitively and at an informal level as follows. Recall that agents need to assess what is the signaling value of interest rates in order to determine the forward yield curve. The signaling value depends on the magnitude of $\hat{\lambda}_t$, but also on the magnitude of $\Delta i_{t,t+1}$.

Consider again two contrasting scenarios. In the first scenario assume that, for sheer illustration, the base rate stood at 475 basis points in the previous period. The Central Bank, concerned for the inflationary outlook, decides that the long-run needs to be equal to a given target, which it can achieve by rising rates by twenty-five basis points relying on the fact that agents shall view such move as a signal that further rate hikes are likely to happen. Hence, the long portion of the yield curve responds to a shift in the base rate by a greater factor than the short-portion of the yield curve so that the Central Bank manages to achieve its initial goal by letting rates be equal to 500 basis points.

Turning attention to the second scenario, assume instead that the short-run nominal rate stood at 450 basis points in the previous period. Does the Central Bank need also under this scenario to bring rates to a level of 500 basis points as in the previous example? If the Central Bank does so, it would engender a change in interest rates of fifty basis points, which would signal to agents that monetary policy shall in future be quite aggressive in hiking interest rates than what agents would have believed had the Central Bank hiked rates by only 25 basis points. Therefore, in this second scenario a level of 500 points for $i_{t,t+1}$ brings about a much larger shift in the long-run portion of the yield curve than in the previous scenario. It then follows that the Central Bank needs a much lower level for the base rate in the second scenario to engender the desired shift in the long-rate. Hence, the higher is $i_{t-1,t}$ the higher $i_{t,t+1}$ needs to be to achieve any given projected rate of inflation.

We develop an important caveat before concluding this section. One could believe that it is quite natural that $i^{**}_{t-1,t}$ feeds upon $i^{**}_{t,t+1}$ since the projection for inflation $n+q$ periods ahead at time t is expected to be quite close to the projection for inflation at time *t+1*. Such remark, however, would bear a misconception. In fact, the term $E_t(\epsilon_{t+n+q})$ (which captures the relevant stochastic factor for the inflationary forecast by the Central Bank) appears in the first order conditions, as shown by (4.3.14) and (4.3.15). Hence, the fact that $i^{**}_{t-1,t}$ contributes in itself to the determination of $i^{**}_{t,t+1}$ cannot depend upon the correlation in the inflationary forecast between the two periods.

The pattern of path-dependence stretches for two periods so that $i^{**}_{t,t+1}$ depends both upon $i^{**}_{t-1,t}$ and $\Delta i^{**}_{t-1,t}$. We now build on the result of this section to clarify how such pattern of path-dependence implies that interest rate smoothing is optimal for the Central Bank.

4.4.3 Optimal Partial Adjustment

Does the model imply that monetary policy is conducted in an inertial way, so that the lagged value of the interest rate is in itself predictive of the current value of interest rates? We show in this section that it is indeed optimal for the Central Bank to adjust interest rates through a partial adjustment mechanism. Therefore, the current optimal level of the nominal interest rate $i^{**}_{t,t+1}$ is, holding other factors constant, increasing both in the level of the lagged nominal interest rate $i^{**}_{t-1,t}$ and in the level of the change in the interest rate $\Delta i^{**}_{t-1,t}$. We articulate this finding in the next proposition, before explaining it by close analogy with the arguments developed in the previous section.

Proposition 4.4.3. ***(Optimal Interest Rate Smoothing both with respect to the Lagged level and the lagged change in rates):*** *The current short-run nominal interest rate $i^{**}_{t,t+1}$ is increasing in both the level of the lag of interest rates $i_{t-1,t}$ and the lagged level of the interest rate change $\Delta i_{t-1,t}$.*

Proof. We aim to show that the marginal cost of increasing $i_{t,t+1}$ (the right hand side of the first order condition of (4.3.13) set to zero at an optimum) is always decreasing in both $i_{t-1,t}$ and $\Delta_{t-1,t}$, so that:

$$\frac{\partial E_t \left\{ \frac{\partial L_t}{\partial i_{t,t+1}} + \beta \frac{\partial L_{t+1}}{\partial i_{t,t+1}} + \sum_{j=0}^{\infty} \beta^j \frac{\partial L_{t+j}}{\partial \alpha_m(\hat{\lambda}_{t+j})} \frac{\partial \alpha_m(\hat{\lambda}_{t+j})}{\partial \hat{\lambda}_{t+j}} \frac{\partial \hat{\lambda}_{t+j}}{\partial \hat{\lambda}_t} \frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}} \right\}}{\partial i_{t-1,t}} < 0; \tag{4.4.3}$$

$$\frac{\partial E_t \left\{ \frac{\partial L_t}{\partial i_{t,t+1}} + \beta \frac{\partial L_{t+1}}{\partial i_{t,t+1}} + \sum_{j=0}^{\infty} \beta^j \frac{\partial L_{t+j}}{\partial \alpha_m(\hat{\lambda}_{t+j})} \frac{\partial \alpha_m(\hat{\lambda}_{t+j})}{\partial \hat{\lambda}_{t+j}} \frac{\partial \hat{\lambda}_{t+j}}{\partial \hat{\lambda}_t} \frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}} \right\}}{\partial \Delta i_{t-1,t}} < 0;$$

Note that the marginal contribution of $i_{t,t+1}$ to $E_t(L_t)$ in equation (4.3.14) is diminishing in $i_{t-1,t}$, since differentiating (4.3.14) with respect to $i_{t-1,t}$ we obtain:

$$\frac{\partial E_t \left(\frac{\partial L_t}{\partial i_{t,t+1}} \right)}{\partial i_{t-1,t}} = -\left(2\beta^{n+q}\gamma^2 \alpha_m(\hat{\lambda}_t) \right) \left(1 + \alpha_m(\hat{\lambda}_t) \right) < 0; \tag{4.4.4}$$

Furthermore, note also that (4.3.15) and (4.3.18) imply that $\frac{\partial E_t(\hat{\lambda}_t)}{\partial i_{t,t+1}}$ and $\frac{dE_t(\hat{\lambda}_{t+j})}{di_{t,t+1}}$ are both increasing in $\Delta i_{t-1,t}$. This, together with Remark 4.3.1 and equation (4.2.12), implies that the following is verified:

$$\frac{\partial E_t \left\{ \sum_{j=0}^{\infty} \beta^j \frac{\partial L_{t+j}}{\partial \alpha_m(\hat{\lambda}_{t+j})} \frac{\partial \alpha_m(\hat{\lambda}_{t+j})}{\partial \hat{\lambda}_{t+j}} \frac{\partial \hat{\lambda}_{t+j}}{\partial \hat{\lambda}_t} \frac{\partial \hat{\lambda}_t}{\partial i_{t,t+1}} \right\}}{\partial \Delta i_{t-1,t}} < 0; \tag{4.4.5}$$

Equations (4.4.5) and (4.4.4) jointly imply that (4.4.3) holds true. This, in turn, means that the as the Central Bank seeks to minimize its loss function is shall set a higher value of $i_{t,t+1}$ the higher $i_{t-1,t}$ and $\Delta i_{t-1,t}$ are. □

The intuition for the result is quite similar to the explanation behind the path-dependency result of Proposition 4.4.2. Two separate effects are at work.

The *reputation effect* induces the Central Bank to serially correlate interest rate changes as to induce agents to revise upwards the parameter $\hat{\lambda}_t$ governing the steepness of the yield curve. Therefore, if $\Delta i_{t-1,t}$ is positive, the Central Bank has, holding other factors constant, an incentive to set a high level for the current nominal rate as to increase $\hat{\lambda}_t$. If it does so, it will expect to face a steeper yield curve in future, which is welfare rising. Such incentive is rising in the magnitude of $\Delta i_{t-1,t}$.

Conversely, if $\Delta i_{t-1,t}$ is negative, the Central Bank has, holding other factors constant, an incentive to set a low value for $i^{**}_{t,t+1}$. In fact, the lower is $i^{**}_{t,t+1}$ in this scenario, the steeper is the future yield curve faced by the Central Bank as a result of agents' upwards revision of the parameter $\hat{\lambda}_t$. Such incentive to set a low value for $i^{**}_{t,t+1}$ is in this case decreasing in $\Delta i_{t-1,t}$.

Therefore, the *reputation effect* gives an incentive to the Central Bank to set the current level of the interest rate in such a way as to increase the historical serial correlation of interest rate changes. This is a first source of partial adjustment in interset rates or inertia, which makes the level of $i^{**}_{t,t+1}$ increasing in $\Delta i_{t-1,t}$.

Note that such effect is particularly strong the higher is δ, the Central Bank's aversion to a high level of the square short-run interest rate. This is so for the Central Bank's incentive to face a steep yield curve is increasing in δ. The more is the Central Bank averse to a very high level for short-run interest rate, the greater welfare gain it can gain from being able to control inflation via minimal changes in the short-run interest rate.

The second effect at work to generate this pattern of inertia in the level of the interest rates is the *yield curve expectation effect*. As previously argued, any level for the long-term interest rate is a function of $i_{t,t+1}, \hat{\lambda}_t$ *and* $\Delta i_{t-1,t}$. If the Central Bank needs to increase the long-term rate, for instance, it can do so with a low level of $i_{t,t+1}$ as long as $\Delta i_{t,t+1}$ is sufficiently large. This is so for it is the term in $\Delta i_{t,t+1}$ which drives the signaling value of interest rates. The higher is $\Delta i_{t,t+1}$, the greater the magnitude of future interest rates. Hence the level need to achieve any target level for the long-term rate $r_{t,t+m}$ is increasing in $i_{t-1,t}$. Hence if $i_{t-1,t}$ is low, even a relatively low level for $i_{t,t+1}$ can ensure that $\Delta i_{t,t+1}$ is positive and large enough to ensure that the yield curve gets steeper as it is required by Central Bank to counter the inflationary shock with a small movement in short-term rates.

Similarly, if the Central Bank needs to lower the long-run rate, it can do so without setting $i_{t,t+1}$ at a very low level as long as $\Delta i_{t,t+1}$ is negative and of sufficiently large absolute sign. If $\hat{\lambda}_t$ is large, it is sufficient for the Central Bank to set $i_{t,t+1}$ at a level slightly lower than $i_{t-1,t}$ to trigger off a large shift in the long portion of the yield curve as agents expect further interest rate cuts to materialize in the near future. Once again, this shows that the level of the nominal interest rate needed to achieve a given target level for the long-term rate is also rising in $i_{t-1,t}$ as a result of the *yield curve expectation effect*.

This discussion motivates the result of Proposition 4.4.3. Summarizing the results of this section, a partial adjustment mechanism for interest rate changes turns out to be optimal because of two effects: i) *the reputation effect* induces the Central Bank to preserve the informativeness of interest rate changes so that the yield curve is steep; ii) *the yield curve expectational effect* which implies that the signaling value of interest rate changes is determined also by $\Delta i_{t,t+1}$ and not only by $i_{t,t+1}$.

4.4.4 How the Marginal Value of Credibility Changes Over the Term of a Central Banker's Mandate

Does a veteran Central Banker face the same pressing incentive to preserve her reputation for serially correlating interest rate changes as a newly appointed Central Banker does? We conclude the analysis of this section by briefly addressing this question.

The answer to this question hinges crucially on how much memory, loosely speaking, agents enjoy when they determine by OLS the magnitude of the parameter $\hat{\lambda}_t$. We specify in the following definition two different regimes for the process by which agents choose what is the relevant sample period to be employed in the computation of $\hat{\lambda}_t$.

Definition 4.4.1. *We say that agents employ a open-window yield curve model if the parameter* $\hat{\lambda}_t$ *is computed according to: (4.2.17).*

Instead, we define agents to follow a closed-window learning process if they calculate $\hat{\lambda}_t$ *according to:*

$$\hat{\lambda}_t = \begin{cases} \frac{\sum_{j=2}^{j=t} \Delta i_{j,j+1} \Delta i_{j-1,j}}{\sum_{j=1}^{j=t} (\Delta i_{j,j+1})^2}; & if\ t \leq T; \\ \frac{\sum_{j=t-T+1}^{j=t} \Delta i_{j,j+1} \Delta i_{j-1,j}}{\sum_{j=t-T+1}^{j=t} (\Delta i_{j,j+1})^2}; & if\ t > T; \end{cases} \tag{4.4.6}$$

Under closed-window yield curve modeling, agents employ only the last T monetary policy decisions to estimate $\hat{\lambda}_t$. Instead, the entire history in office of the Central Banker is taken into account upon computing $\hat{\lambda}_t$ under an open-window mechanism.

Having fixed ideas with this definition allows us to study how the incentives faced by the Central Banker vary with the length of her tenure in office under each mechanism for the determination of λ_t. We turn attention to this task in the next Proposition.

Proposition 4.4.4. ***(Reputation Effect Stronger for a Recently Appointed Central Banker):***

If learning happens through an open-window process: the magnitude of the reputation effect by which the Central Banker wishes to serially correlate interest rate changes to face a steeper yield curve is diminishing in the length of the history in office of the Central Banker.

If, instead, learning happens through a closed-window process with a window of size T: the magnitude of the reputation effect diminishes over the length of the history in office for the Central Banker as long as the history in office for the Central Banker is shorter than T periods; once the history in office exceeds the T periods threshold, then the magnitude of the reputation effect is no longer expected to be a function of the length of the history in office of the Central Banker.

Proof. We first proof the first part of the proposition which applies under an open-window mechanism.

Note that under the open-window assumption:

$$\frac{\partial}{\partial t+s}\left(\frac{\partial E_t(\hat{\lambda}_{t+s})}{\partial \Delta i_{t+s,t+s+1}}\right) = \frac{\partial}{\partial t+s}\left\{\frac{\partial E_t\left\{\frac{\sum_{j=1}^{j=t+s} \Delta i_{j,j+1} \Delta i_{j-1,j}}{\sum_{j=1}^{j=t+s} (\Delta i_{j,j+1})^2}\right\}}{\partial \Delta i_{t+s,t+s+1}}\right\} < 0; \tag{4.4.7}$$

This is so for as the number of observations grows larger, each single observation for the rate of serial correlation has an increasingly smaller impact upon the OLS estimate $\hat{\lambda}_{t+s}$ since the denominator of (4.2.17) is increasing in t+s.

The observation above still applies to a closed-window scenario as long as $t+s<T$. However, in a closed-window scenario the size of the sample is fixed to be equal to T once the threshold T is achieved. Hence, after the threshold is achieved, the length of history in office of the Central Banker no longer affects the size of the sample agents use to calculate by OLS the coefficient of serial correlation of interest rate changes. This is so for after the threshold T is achieved the historical length of the tenure in office of the Central Banker does not impact the expectation of the denominator of (4.4.7) so that the expected impact of $\Delta i_{t,t+1}$ on $E_t(\hat{\lambda}_{t+s})$ would not be diminishing in t+s. □

The proposition implies that partially different conclusions apply in each regime. The *reputation incentive* drops in both regimes over time from the date of appointment to time T. This implies that a newly appointed Central Banker, who finds it easier to induce agents to revise their estimate of $\hat{\lambda}_t$, has a particularly strong incentive to serially correlate interest rate changes.

However, the two regimes have different implications after the Central Banker has been in office for more than T periods. The closed-window regime implies that at this stage the *reputation incentive* is not expected to vary over time. The intuition behind this result being that in the closed-window regime the sample size used to compute $\hat{\lambda}_t$ does not vary after the threshold T is reached.

On the other hand, the open-window regime implies that the marginal impact of monetary policy on $\hat{\lambda}_t$ drops steadily across time.

We believe that in a more realistic setting agents might attach a greater weight to most recent observations that they do to long dated ones. This could be so as agents believe that regime shifts and structural breaks are pervasive in their yield curve model. Hence agents believe also that that the recent history is more predictive of the future that long-dated observations are. Under this light, we regard the closed-window model as being somewhat more realistic. We therefore prefer to use this setting in drawing our final implications, to which we now turn attention.

4.5 Conclusions

We are now ready to study the implications of the model for a number of important questions central to the interest rate smoothing literature. First of all, what does the model imply in reference to the often stated claim that Central Banks act too little and too late?

This issue can be clarified, we have argued, by noting that a timid response in the short-term rate does not necessarily imply a timid response in the measure of monetary policy which drives macroeconomic fundamentals. In fact, if the Central Bank has been historically observed to carry out monetary policy via a partial adjustment mechanism, a small change in the short-term rate might lead to a large shift in the medium and long portion of the yield curve (a more useful indicator of the monetary policy stance than short-term rates are).

The Central Bank is able to shift the medium and the long portion of the yield curve with a small shift in short-term rates as long as agents have learnt that historically the Central Bank conducts monetary policy in a fashion that generates a low reversals to continuations ratio. In fact, agents have to determine what is the signaling value of a change in the short-term rate. If the Central Bank has been historically known to serially correlate interest rate changes and to deliver low reversals to continuations ratios, agents attach a great signaling value to any observed change in the short-term rates and hence revise their forecasts for future forward rates by a much greater magnitude than the observed change in the short-term rate. This is so for, if $\hat{\lambda}_t$ is high, the Central Bank is believed to follow the latest change in the base rate with a series of further moves in the same direction.

The Central Bank does not need to commit to a given interest rate smoothing rule. Rather, we interpret the credibility of monetary policy as the historical record the Central Banker enjoys with respect to the infrequency of reversals in the trend for short-rate rates. Therefore, our model works in a discretion framework rather than in a commitment one.

Why is it optimal for the Central Bank to be able to effect a large change in medium and short-term rates by moving short-term rates initially by only a small amount? This stems from the assumption that the quadratic loss function for the Central Bank is increasing in both the level of projected inflation and in the level of short-term interest rates. Observe that the short-run rate does not need to jump immediately to a very high value if, for instance, a large inflationary shock hits the economy and the Central Bank enjoys the reputation of being an interest rate smoother. If this is the case, even a gentle increase in interest rates acts to drive long-run rates to a level that keeps inflation in check. Instead, if the Central Bank cannot bring into effect a large movement in the long portion of the yield curve with a small movement in short-run rates, then the short-run rate shall have to initially rise by a large amount. This is an undesirable outcome for the Central Bank since the quadratic level of the short-run rate contributes to the loss function.

It could be observed that, on the other hand, if long rates are very responsive to changes in short-run rates then the Central Bank cannot lower interest rates quickly whenever inflation looks tamed. However, the quadratic form of the loss function implies that the Central Bank prefers a scenario in which interest rates never get too high or too low for a long time to one in which interest rates can take a potentially very high or very low value for prolonged periods of time.

Note also that our analysis is quite different to assuming directly that the Central Bank regards interest rate smoothing as an objective in itself. In fact, the assumption that the Central Bank wants to smooth interest rates would imply that it is trying to stabilize the short-run rate around its current level. However, in our model the Central Bank simply prefers, holding other factors constant, that the square level for short-run rates be as low possible and no explicit interest rate smoothing objective is postulated.

Why do interest rates exhibit short-run path dependence and a partial adjustment mechanism? Two channels operate to generate these results. First, the Central Bank needs to preserve its reputation for not reversing the direction of interest rate changes too frequently and for implementing interest rate changes via a series of positively serially correlated interest rate changes. Without such reputation for partial adjustment, the Central Bank would be unable, in spite of a minimal movement in short-run rates, to lean aggressively against the wind of an inflationary shock by bringing about a large movement in long-term rates. This is the mechanism we defined as the *the reputation effect* in the main text.

A second mechanism that generates partial a positive correlation between the current nominal short-run rate and its lag consists of what we termed as *the yield expectation effect*. The higher the lagged level of interest rates, the higher the current level at which interest rates need to be in order to achieve a given target level for long-run rates. This is so for agents use the change in the short-term rate, rather than the level of the short-term rate itself, in order to assess the signaling value of interest rates. For instance, as soon as the current monthly interest rate drops below the lagged one, regardless of how high the current short-run rate is, the yield curve in our model inverts and the yield on long-term maturities falls below the yield on short-term bills.

The combination of the *reputation effect* and the *yield expectation one* acts to generate not only a partial adjustment mechanism, but also drives a pattern of short-run time-dependence in the model. In fact, we have shown that he current interest rate is increasing both in the level of the lagged rate and in the in the rate of change of its lag. The reputation effect, in fact, makes it advantageous for the Central Bank to try to increase its credibility by building a record for serially correlating interest rate changes. On the other hand, the yield expectation effect suggests that the medium and long-term interest rates are also determined by the rate of change of the current short-term rate (and not only by its current level).

Finally, does the model imply that a newly established Central Banker has a particularly strong incentive to serially correlate interest rate changes? We have shown that this is the case in the model of this chapter. However, there is a time threshold after which a recently appointed Central Banker faces the same incentives as a veteran Central Banker under our favored closed-window mechanism (a mechanism whereby only the most recent observations of monetary policy actions are used by agents to formulate their forward rates yield curve models).

Before proceeding to draw some tentative policy implications, a number of caveats must be developed. First of all, we have assumed by simplicity that agents estimate $\hat{\lambda}_t$ by looking at the monthly rate of serial correlation of interest rate changes. In practice, agents might adopt a more complicated rule, and use a richer auto-regressive model to estimate forward yields. Though we have not explicitly shown this, such extension would not seem to alter the qualitative implications of the model.

Secondly, it is plausible that agents might attach a greater signaling value to interest rate changes at the turning points of monetary policy. Therefore, when interest rates display a reversal, forward yields should be revised by a much greater extent than when interest rates, instead, exhibit one more continuation movement in the same direction as the previous one. To account for this observation would only complicate the model but does not seem to alter its main results.

A third important qualification is also in order. The long-run forward rate of interest should really be held to be exogenous, if, for instance, we take a Ramsey model as a benchmark in which the interest rate is linked to the marginal product of capital. And, in turn the marginal product of capital is driven in the steady state by agents' rate of inter-temporal discount. Therefore, monetary policy should be able to have a signaling impact only on a relatively short portion of the forward yield curve.

Nonetheless, long-term rates, which in a term structure framework are also driven by the short and medium portion of the yield curve, should still be responsive to monetary policy. Note that it is often observed that in practice the most volatile yield is the one on the two-years bond rather than the one on ten years Treasuries so that long-run rates should be less volatile than short-run ones.

We can at this stage turn attention to some policy implications of the model of this chapter. We answer this question with a word of caution. Our framework is quite restrictive. Therefore we can only highlight what the policy implications are for the specific effect we have studied, which might contrast with the lesson delivered by other important effects we have omitted.

However, a number of observations emerge. Partial adjustment does not necessarily imply that the Central Bank is too timid in leaning against the wind of inflationary shocks. For, we have argued, the crucial indicator of the actual monetary policy stance lies in the shape of the medium and long portion of the yield curve. If agents understand the partial adjustment mechanism employed by the Central Bank, financial markets ensure that the Central Banks' apparently timid response to a shock translates into an actually aggressive one. To this effect, the Central Bank should never invert the direction of interest rate changes too aggressively and should ensure that it develops a reputation for carrying out a path of interest rate smoothing.

The Central Banker's aversion to reversals in interest rate setting does not stem from a pattern of personal pride in the context of our model. Rather, it represents an optimal strategy to ensure that the signaling value of interest rate changes is preserved.

Finally, Central Bankers do not need in the context of our model to commit to a given mechanism for the partial adjustment of interest rates. The beneficial effects stemming for adopting a partial adjustment mechanism for the setting of interest rates highlighted by our framework apply under discretion as long as agents use a learning and adaptive model to assess how strong is the signaling value of interest rate changes.

However, Central Bankers have no incentive in the model of this chapter to be secretive. One of the implications of the model is that it could actually be ideal for the policy-makers to signal to agents their *non-binding* forecasts for the future path of short-run rates. This shall enable agents to understand what is the right signaling value they should attach to interest rate changes, enhancing the responsiveness of the medium portion of the yield curve to changes in short-term rates, which, this chapter argues, is an important factor policy-makers need to consider in the setting of monetary policy.

Chapter 5

Conclusions and Final Discussion

We would like to describe the research strategy pursued in this conclusive chapter with an analogy often employed. We regard each model we have developed in the central chapters of the thesis as a very specialized exercise designed to understand a specific effect rather than to provide a general theory. Hence each model aspires to stand, however minor and modest its contribution might be, to the large body of monetary economics theory the way a small point in space stands to an atlas. To illustrate a number of research questions, the atlas of economic theory must be browsed drawing to find the insights of a number of its geographical points that are relevant to the issues at hand. This is what we attempt to do in this conclusion: on the one hand we attempt to highlight the insights the models we have developed seek to contribute to each research question; on the other hand, we develop some qualifications about our findings that are necessary since the analytical frameworks we have investigated are not general models.

5.1 Implications For Voting Secrecy in a Monetary Union

We start this final discussion with the research question investigated in Chapter 2: Should the voting records of individual members of the Interest Rate Setting Panel of a Monetary Union be published? The considerations we have developed in the analysis are valid only if we take at face value the ECB's claim that the publication of individual voting records forces *partisan interests* to take priority over any other consideration. However, even if this assumption is taken at face value we find that Voting Secrecy is not unambiguously optimal.

At a first level of the analysis, we provide the following conclusion: if the structure of supply shocks is held to be exogenous, as shown by Proposition 2.2.1, then Voting Secrecy is welfare optimal for the Monetary Union as a whole as it decreases macroeconomic volatility. This result is perhaps quite trivial and intuitive. It rests on the intuition that Voting Secrecy acts in this scenario as an insurance policy by ensuring that the Central Bank of the Monetary Union stabilizes macroeconomic fundamentals even in a region hit by asymmetric shocks.

If Voting Secrecy is welfare rising in this context for the Monetary Union as a whole, would it also be welfare rising for each individual region? This question is particularly interesting for the welfare analysis of Voting Secrecy in the Center. In fact, we show that the Center under Voting Transparency is the most likely median voter and hence almost invariably would get its first best choice under Voting Transparency. However, we show in Proposition 2.2.2 that Voting Secrecy is also welfare rising for the Center when the two peripheral regions are equally asymmetric to the Center. In fact, the Center faces under Voting Transparency a sufficiently high probability of being out-voted in spite of being ex-ante the most likely median voter. Therefore the Center prefers getting the insurance policy of Secret Voting rather than getting under Voting Transparency its first best choice in most case while being sharply-outvoted in some rare, but very welfare costly, contingencies.

This result is dependent on the specific setup of the model we have developed and on the number of member countries introduced in the Monetary Union. In fact, we conjecture that the probability that the Center experiences an asymmetric output supply shock diminishes as the number of different industries in the Monetary Union in our model grows larger. Hence, this second result might not robust to extensions. However, the notion that the Center itself, despite being the most likely median voter under Voting Transparency, might prefer to surrender its likely median voter position by choosing Voting Secrecy seems interesting. This second result could furthermore explain why indeed a Voting Secrecy arrangement has been favored for the ECB.

It might also be conjectured, following Buiter's remarks (Buiter 1999), that the arrangement of Voting Secrecy tends to put a disproportionate power in the hands of the President of the Interest Rate Setting Panel. If this is the case, the Center might prefer Voting Secrecy as it minimizes the risk that peripheral countries could hold any substantial weight in determining monetary policy. This observation holds true as long as the Center has full control over the appointment of the Central Bank's President. Note that, however, the inherent secrecy of the European Central Bank's makes this remark difficult to test.

Holding the structure of the supply exogenous, is the Center better off with Voting Secrecy in all scenarios? We answer this question in the negative in Proposition 2.2.3 in a scenario in which one of the two peripheral regions enjoys an industrial structure sufficiently similar to the one existing in the Center. To understand this result one can think about the limit case in which two countries in the model have an identical industrial structure. We have labeled this scenario as the *two Centers-one Periphery case*. In this scenario, output supply shocks in the two Central Regions are identical. Hence, the Center is always better off with Transparent Voting which ensures that the its partisan interests prevail in all cases.

The results of this first level of the analysis can be reversed if we let the pattern of industrial structure be endogenous to the monetary policy arrangement chosen. We do so following a line of investigation originally pursued by Krugman (Krugman 1991).

Why would the pattern of industrial structure be endogenous to the choice of the voting regime in our model? We show in Remark 2.3.1 that Voting Transparency implies a higher degree of macroeconomic volatility in each region relative to the Voting Secrecy case, which seems to be in line with the European's Central Bank concerns. However, the results of our general equilibrium model find that such increase in macroeconomic volatility might provide each firm with an incentive to locate widely in various regions rather than concentrating its productive activities in only one region of the Monetary Union.

What is the effect of widespread industrial location? Supply shocks become more symmetric across regions of a Monetary Union, as noted by Krugman (Krugman 1991), if firms locate production widely rather than narrowly. This observation is particularly important if we bear in mind that Proposition 2.3.1 implies that Transparent Voting induces a more symmetric industrial structure across the various regions of the Monetary Union relative to Secrecy Voting. Hence, Transparent Voting might induce supply shocks to be more symmetric relative to the Secret Voting case. For this reason Transparent Voting is not necessarily welfare diminishing when the issue of industrial location is held to be endogenous to monetary policy.

Note that our results are suggestive of a theoretical effect, but the strength of this effect in practice still needs to be empirically assessed. Krugman finds a suggestive finding that geographic specialization is much stronger in the US than in Europe (Krugman 1991). However, this suggestive finding implies that each industry tends to diversify its geographical location as the barriers to trade become lower. This effect is not the same as the one we analyze in Chapter 2. In fact, for our analysis to be empirically relevant we need to verify that each industry tends to geographically specialize as volatility in each region of the Monetary Union increases. No empirical testing of this effect has been to date carried out and hence the empirical relevance of the results of the second model of Chapter 2 is still to be established.

A number of theoretical observations can also be advanced in order to qualify our findings. First of all, there is no financial market in the model we have developed in Chapter 2. This is counter-factual as in reality firms can hedge macroeconomic volatility via a number of financial instruments.

However, we would like to argue that the hedging of fluctuations in the level of aggregate demand does not seem easily viable. In fact, unlike in a exchange rate hedging transaction, the party which would assume the risk of fluctuations in aggregate demand has no instrument to hedge such risk in its turn. It can be recalled that a party selling, for illustration, a put option on the dollar can always short the currency to carry out delta-hedging and hence lock in the derivative premium (Bjork 1998). Instead, a party that hypothetically sells a derivative on the level of aggregate demand is unable to find a perfect hedge for such risk. For this reason, the hedging of aggregate demand risk is, if at all possible, extremely expensive.

Secondly, is the assumption by the European Central Bank that upholding secret individual voting records ensures that members of the Interest Rates Setting Panel are insulated from partisan pressure really tenable at face value as we have chosen to do in Chapter 2? Buiter's observation that individual voting records are at least *observable* in practice, even if not in practice *verifiable*, probably holds some force.

However, the sheer fact that such actions are verifiable implies that no official discussion of individual voting records can be held. Nor can the national media easily scrutinize the behavior of a given voting member of the Interest Rate setting panel, who the ECB assumes would be a mere champion of national interest under Transparent Voting. Therefore the lack of verifiability of Central Banker's individual voting records in a Monetary Union might hold at least some weight in insulating policy-makers from partisan pressure.

However, a second objection can be raised to the ECB's assumption. What is the driving force that renders a given policy-maker dependent on the interests of the country that has appointed her in office? Let us hypothesize that such prominence of partisan interests might be due to the fact that policy-makers might fear that they risk not being re-elected in any kind of office ever again if they displease the country that has originally appointed them. Such assumption holds policy-makers to be fully self-interested *homini oeconomici*. But even if we accept such assumption at face value, it can be argued that an *homo oeconomicus* be insulated from partisan pressure through a sufficiently high guaranteed remuneration after his term in office is over. However, we feel that even such belief might be simplistic as a variety of considerations might motivate policy-makers.

Therefore, we content ourselves with just analyzing some of the theoretical implications of the ECB's statement and recognizing that it might be difficult to test the plausibility of Issing's (Issing 1999) statement that Transparent Voting would render partisan interests impossible to resist.

Some other off-model considerations might be worth investigating. First of all, the process of transparency might allow policy-makers to receive a more detailed feedback from the competent public. Hence, it might be argued that Secret Voting might diminish the public's ability to contribute constructively to the debate on the optimal conduct of monetary policy. However, probably this point should not be over-emphasized for the public is certainly informed about the consensus decision of the Central Bank which might explain some details about the policy decision at a press conference, as the ECB does.

5.2 Voting Secrecy and the Behavior of the Long-Run Portion of the Yield Curve

The analysis of Chapter 4 also yields a suggestive implication for Voting Secrecy. Note, in fact, that the publication of individual voting records has the important effect of rendering future policy decisions more predictable. In fact, observers are aware that, for instance, the December 2001 meeting of the Bank of England ended in a 7-2 vote in favor of holding rates on hold as opposed to carrying out a 25 basis points decrease in the repo rate. A precise knowledge of the number of voters in favor of a policy action different to the one eventually implemented allows agents to gain insight on future policy decisions. For example, had four members of the Committee been in the minority party rather than two, the public would have attached a higher subjective probability of a twenty five basis points decrease in the base rate at the next meeting of the MPC.

Why should a Central Bank be averse to surprising agents and, instead, should seek to be predictable? An unpredictable Central Bank, we argue, might find it difficult to trigger off a large movement in the medium portion of the yield curve with a small movement in the short portion. This is so for the yield curve model developed in Chapter 4 delivers long-term rates that are responsive to monetary policy only if agents feel capable of extracting a signal on the future conduct of monetary policy from the current interest rate decision. We conjecture the off-model consideration that secrecy voting, and to some extent information secrecy too, renders the relationship between the short-end and the long-end of the yield curve quite an uncertain one and hence might diminish the Central Bank's ability to affect large movements in the medium portion of the yield curve with a small initial change in short-term yields.

5.3 Implications for Gradualism and Partial Adjustment

Having explored the consequences of Voting Secrecy in a Monetary Union, we turn attention to the implications of information secrecy, studied in Chapter 3, and of the partial adjustment model for interest rates changes studied in Chapter 4. The models developed in these two chapters can both yield an insight into the following question: Why do Central Banks tend to respond to macroeconomic news through an initially timid and gradual response so that, as documented in Section 1.3, Central Bankers are often accused of doing *too little too late* (Goodhart 1997)?

The model of Chapter 3 implies that Central Bankers are cautious in responding aggressively to macroeconomic news for they fear this would affect consumer's confidence in a pro-cyclical manner. A statement to this effect was recently uttered by the ECB's Chairman Duisenberg (Duisenberg 2001). In fact, the signaling game we have modeled in Chapter 3 shows that agents try to infer the Central Banker's private information through the Central Bank's actions. If interest rates, for instance, are lowered very abruptly agents' consumer confidence might plummet. The analysis of this chapter, therefore, is narrowly focused on the interaction between monetary policy and *the animal spirits* of the agents although we narrowly define such animal spirits as consisting solely of the agents' assessment of their expected future disposable income.

We show that whenever agents' place a large weight on their capital income in determining their consumption plans Central Bankers might have an incentive to stabilize agents' expectations and let interest rates be less volatile under asymmetric information than they would be in a perfect information scenario, as shown by Proposition 3.4.1. Therefore, this model seems to formalize some suggestive comments put forward in the conclusive section of an influential survey (Clarida, Gali, and Gertler 1999) in which it is suggested that interest rate smoothing might be the result of the Central Bank's efforts *to stabilize financial markets.*

A number of qualifications are in order. First of all, the empirical relevance of this effect, notwithstanding Duisenberg's remarks, is difficult to test. At a first level this is so for natural experiments consisting of a change of base rates of over a hundred basis points are not commonly observed in OECD countries. Secondly, an innovation in monetary policy can give rise to a variety of different informational effects. Agents might not update their beliefs on their future capital income if they think that a given decrease in interest rates simply reflects a revision in the expected rate of inflation, without any implication for other factors. However, if instead agents believe that the sudden loosening of monetary policy signals great weakness in the projected rate of growth, then an aggressive change in interest rates monetary policy innovation might trigger off, as in our model, a large pro-cyclical movement in consumer's confidence.

Furthermore, the informational advantage of the Central Bank might be time-varying. One might conjecture that the informational advantage of the Central Bank might be larger at turning points than at any other phase in the cycle, even though such statement has not been tested in the study of David and Christina Romer (Romer and Romer 2000). Hence, the reaction of consumers' confidence to monetary policy might also vary depending whether the business cycle stands at a turning point or not.

In the context of the model developed in Chapter 3,gradualism in monetary policy might not imply a loss in welfare since it helps stabilizing consumers' confidence when Proposition 3.4.4 holds. This statement is subject to a further important qualification. Assume that the Central Bank stabilizes consumers' confidence by not sending signals on possible future macroeconomic shocks that might destabilize consumers' confidence. Then agents are unable to smooth consumption and, as suggested by Joseph to the Pharaoh, save during the seven years of fat cows to spend during the seven years of slim ones.

In the completely different setting of Chapter 4 we also find that gradualism in monetary policy does not necessarily diminish the Central Bank's ability to control inflation and is welfare rising. In fact, we show in Proposition 4.4.1 that the medium and the short portion of the yield curve can react by a large shift even to a small change in short-term rates. The more the Central Bank is known to conduct monetary policy via a number of *continuations* (defined by Goodhart (Goodhart 1997) as serially correlated interest rate changes), the more responsive are medium-term and long-term rates to short-term ones.

We show in the analysis of Chapter 4 that the ability to affect a large movement in the long-end of the yield curve with a small change in interest rates is one Central Banks should treasure. Hence, a partial adjustment behavior for interest rates is optimal in the context of the analysis of Chapter 4. This is so for we have assumed that the quadratic loss function of the Central Bank is increasing in the level of the short-term rate for a number of reasons we have provided. Hence, the fact that long-run interest rates are very reactive to a small change in the short-run rate implies that the Central Bank can effectively stabilize a large inflationary shock with a minimal initial hike in short-run interest rates. This property, which holds if the Central Bank is concerned about the quadratic level of the short-run rate, implies that policy-makers are keen to preserve their capability of affecting long-run rates by a large amount with only a small initial movement in short-run rates. In this way, policy-makers can decrease the volatility of short-run rates.

Note that we have not assumed that interest rate smoothing is an explicit objective of monetary policy. In fact, we have not assumed that the Central Bank seeks to peg the short-run rate at its current level. Instead, the assumption that a quadratic level for the short-term rate enters the loss function implies that the Central Bank faces a great welfare loss if short-run rates are high.

This mechanism relies on the crucial assumption (central to all the models of this family of the literature) that the level for the short-run interest rate enters the Central Bank's quadratic loss function. In fact, such assumption is also crucial in the work of Woodford (Woodford 1999), which proves in the context of a much more sophisticated commitment framework results bearing some analogies to the ones we have derived under a discretion regime.

One aspect of the results of Chapter 4, however, holds even if the Central Bank is not concerned about the level of the short-term rate. In fact, the simple learning yield curve model developed in Proposition 4.2.1 does not rely upon the functional form of the Central Bank's loss function. The result of Proposition 4.2.1 implies in the context of our chosen forward yield model that the higher is the continuations to total changes ratio, the higher is the responsiveness of long-term rate to changes in the short-term rate. As Greenspan reminded Congress in a recent testimony (Greenspan 2001) monetary policy is ineffective if long-run rates do not respond to short-run ones.

Hence we regard three ideas from Chapter 4 to be quite robust: i) the medium-run and the long-run rate should be considered as the real indicators of the monetary policy stance; instead, the current short-run rate, which the Central Bank controls, matters mainly for its signaling value in determining the shape of the yield curve rather than being important in itself; ii) the practice of interest rate smoothing and of conducting monetary policy with a low reversals to total changes ratio allows Central Banks to ensure that long-run rates are relatively responsive even to a small movement in short-run ones; iii) the smoothness in the short-run rate does not imply, at least at a theoretical level, that Central Banks are necessarily overly timid as suggested by a number of observers (see the discussions in, *inter alia*, Goodhart (Goodhart 1997), Ball (Ball 1999) and Rudebusch (Rudebusch 1998)).

Note, however, that all models in the interest rate smoothing literature suffer from a lack of robustness. Explanations for interest rate smoothing behavior based on model uncertainty as in Brainard (Brainard 1967) rely on a restriction on the sign of the third derivative of the loss function. Moreover, the Central Bank can refine their knowledge of the coefficients of the model after implementing a change in interest rates only with a lag of several months. This is so for it takes several months before monetary policy feeds upon output and especially inflation. Hence, models in this family cannot explain why continuations are observed so frequently at such close intervals.

On the other hand, explanations based on data uncertainty, as observed by Sack and Wieland (Sack and Wieland 2000), can only explain why a large innovation in the data or in forecasts is not followed by a large innovation in interest rates. But this is not sufficient to produce partial adjustment behavior (Sack and Wieland 2000) as models in this family predict the same interest rate path as would hold under the certainty equivalence case.

These observations on the lack of robustness of all families of interest rate smoothing model serve to highlight how no single model is sufficient to account for the all the stylized facts motivating the interest rate smoothing literaure. Rather, we believe that the practice of interest rate smoothing is justified by a wealth of mutually complementary accounts than can only jointly explain this important feature of monetary policy.

5.4 Implications for the Welfare Analysis of Information Transparency

Should Central Banks, when endowed with asymmetric information, be transparent about their macroeconomic predictions? Or, rather, the FED's argument that full information disclosure would induce excessive volatility in financial markets holds some force (Goodfriend 1991)? We address this question drawing on some insights from the analysis of Chapter 3.

The answer to this question is not unambiguous in the setting we develop. In fact, we show in Proposition 3.4.4 that information secrecy might be welfare rising if agents put a sufficiently high weight on their expected capital income when determining consumption plans. This is so for a counter-cyclical monetary policy risks triggering off some very large pro-cyclical wealth effects. This insight of the model is in line with a comment recently uttered by the ECB's Chairman Duisenberg (Duisenberg 2001) trying to justify the passive stance taken by the EBC.

Furthermore, note that if information secrecy is best upheld under Voting Secrecy it would result that Voting Secrecy has also the effect of dampening fluctuations in financial markets.

Note that even this insight is not a general one. In fact, Proposition 3.4.5 shows that information secrecy can be welfare diminishing when a limit pricing outcome obtains in the signaling game we model. This so for for limit pricing behavior under asymmetric information might force Central Banks to undertake costly actions with the only purpose of signaling their type to agents. This inherent inefficiency of limit pricing behavior can make information secrecy suboptimal in our model whenever a totally separating equilibrium with limit pricing holds. Hence Central Bankers might want under certain conditions to share the asymmetric information they are endowed with in order not to have to undertake costly signaling actions when agents learn the Central Bank's information from the path of interest rates. In this case, information secrecy is undesirable and so is voting secrecy (which effectively increases the degree of information secrecy).

However, other elements might make information transparency optimal. One of this effects suggesting stems from the analysis of Chapter 4.

5.5 The Tension between Secrecy and Forecastability

The models developed in this thesis suggest, at least in our highly specialized frameworks, an interesting tension between transparency and secrecy. In fact, on the one hand, the analysis of Chapter 4 shows that a Central Bank has an incentive to be transparent and to allow agents to forecast its actions; on the other hand, the analysis of Chapter 3 suggests that under some stated conditions it is optimal for the Central Bank to retain some secrecy.

In fact, note that it is crucial for the mechanism described in Chapter 4 that agents be able to extrapolate the current action of the Central Bank into the future when they determine forward rates for long-run yields to be sensitive to short-run ones. Were future actions of the Central Banker unpredictable, then it would be difficult for the long-end of the yield curve to price in future interest rate movements in a forward looking manner. This observation is suggestive of the importance Central Banks might attach to being predictable.

It is plausible that the more a Central Bank is transparent about its information, the easier it would be for agents to predict the future behavior of interest rates. However is this remark sufficient to conclude that information transparency is desirable ? Our results yield an ambiguous answer to this question.

In fact, the analysis of Chapter 3 delivers ambiguous implications for the welfare comparison between information secrecy and information transparency. As previously stated, we find that information secrecy is welfare optimal when consumers' confidence carries a very large weight in determining aggregate demand, as outlined in Proposition 3.4.4; however, information transparency is found to be preferable if consumer's confidence has a sufficiently low weight in the determination of aggregate demand so that limit pricing behavior occurs as stated in Proposition 3.4.5. The intuition behind this result rests on the inherent inefficiency of limit pricing, which forces agents to carry out some costly action only to let receivers understand their types as shown, in a microeconomic context, by Milgrom and Roberts (Milgrom and Roberts 1982).

Hence a dilemma arises in the cases in which information secrecy is welfare rising in the context of Chapter 3. In fact, while the results of Chapter 4 indicate that it is optimal for the Central Bank to be transparent and predictable, the results of Chapter 3 state that, if the conditions under which Proposition 3.4.4 holds are verified, secrecy might hold a welfare rising impact since it allows monetary policy not to impart excessive volatility to consumers' confidence.

When should a Central Banker be particularly concerned that a very large monetary policy move can destabilize agents expectations? The results of Chapter 3 suggest that this is likely to be the case when agents exposure to equity markets is large while investment is not responsive to changes in interest rates. Hence, we might suggest that the stabilization of agents expectations is particularly important in the USA economy characterized by widespread equity ownership. One might be tempted to conclude that this observation might also suggest a rationale behind the fact that the FED has opted for an information secrecy arrangement. However, this finding is only a suggestive one for the results of the model of Chapter 3 have purely a qualitative interpretation and lack any empirical testing.

5.6 Implications for Possible Limit Pricing Behavior

However, the model of Chapter 3 can be used to study a variety of interesting questions beyond the welfare optimality of information secrecy. Can, for instance, the quoted excerpt from the MPC's meeting of November 1998 presented in Section 1.4 be suggestive of occasional limit pricing considerations that can be rationalized in the analysis of our model? We answer this question in the affirmative in Proposition 3.4.3 by showing that indeed limit pricing behavior applies in the model of Chapter 3.

It might be objected that, if the Central Bank finds itself sub-optimally constrained by limit pricing behavior, it can always switch to a regime of full information transparency. The very quote we have presented from the November 1998 minutes ((Bank of England 1998), point 36), however, casts some doubts on this point. In fact, it will be recalled that in this occasion some members of the MPC feared leading agents to panic by implementing a seventy-five basis points decrease in the base rate, *notwithstanding the opportunity to explain the information the Central Bank was reacting to in the next*

Inflation Report. This point just highlights that agents might always place some informational weight on the actions of the Central Bank even if the Central Bank fully shares with agents all the available macroeconomic information. This might be so as agents seek to deduce the Central Banker's private information from her actions even when information is transparent. In fact, the assessment of a vast array of different data is a difficult process and agents might learn from the Central Bank's actions how to synthetize into a directional view the information available to them.

5.7 Implications for the Effects of the Publication of Detailed Minutes

Bearing this important observation in mind, we could qualify the implications of the model for the effects of publishing detailed notes for the meetings of the Central Bank. In fact, we have shown in Proposition 3.4.6 that the publication of detailed minutes of the Central Bank's meetings should increase the probability that interest rates are changed while also increasing the magnitude by which they are adjusted when the Central Bank does not keep them on hold. This result rests on the fact that the publication of detailed minutes serves to inform agents about the macroeconomic information observed by the Central Bank regardless of the monetary policy action undertaken. However, we have just observed that notwithstanding full information transparency agents might still deem the Central Banks' action to carry some signaling value as to the Central Banker's assessment of the macroeconomic cycle; the higher the weight this remark carries, the less relevant is the result of Proposition 3.4.6 stating that interest rates are changed more often and by a greater magnitude when the Central Bankers are mandated to publish detailed minutes of their meetings. In fact, the incentive to play a pooling equilibrium in the signaling game of Chapter 3 does not fully dissipate when minutes are published if the actions by the Central Banker carry some informative value even under full disclosure of the minutes.

5.8 Further Implications of Information Secrecy for the Low Reversals to Total Changes Ratio

Is the microeconomic assumption that consumers' confidence could be updated upon observing the latest monetary policy action by the Central Bank capable of biasing the ratio between continuations and reversals in favor of continuations? We answer this question in the affirmative in Conjecture 3.4.9. The intuition for this result lies in the fact that the informational advantage by the Central Bank is assumed to dissipate over time. Hence, we show that an initial timid response to a forecasted inflationary shock is followed by a more aggressive loosening of monetary policy once the shock becomes of public knowledge. This is so for as the informational advantage by the Central Bank gradually dissipates over time so does the incentive not to trigger off pro-cyclical wealth effects by implementing an aggressive monetary policy move. As a result, continuations tend to be more frequent than reversals in the example provided.

However, the conjecture is based upon a simple example in which the informational advantage of the Central Bank lasts for only one period and dissipates very abruptly. The assumption of a more gradual dissolution of the informational advantage by the Central Bank could make the results richer and span the cycle of likely continuations to a longer horizon. However, the model becomes quite involved to solve once we let the informational advantage last for more than one period.

5.9 Further Observations on the Steepness of the Yield Curve and Transparency

A final criticism to the analysis of Chapter 3 stems from one of the results of Chapter 4. In fact, the model of Chapter 3 does not concern itself with long-run rates but assumes that the short-run rate is in itself able to drive economic fundamentals. We would like to argue that, in this context, this simplifying assumption, common to most of the models in the literature, may not compromise the generality of the results as long as a movement in the short-run rates translates itself into a proportional change in long-term yields.

However, this observation suggests an interesting avenue of investigation. Are long-term rates more responsive to short-run ones when the Central Bank fully divulges the information it is endowed with to economic agents and restrains from playing pooling equilibria strategies in the context of Chapter 3 so that agents can extract all the available information from monetary policy? If this working hypothesis were to be true, then information transparency might have some further welfare rising effect which the literature has not yet investigated.

5.10 A Final Thought

Having outlined some implications of the models developed as well as some aspects that seem to lack robustness, we would like to conclude by highlighting the relevance of thinking about the interest rates as being an informational vehicle. If economics differs from physics for the laws it seeks to study are not time-independent but, instead, continuously change according to the policy-rule adopted, then Central Bankers have to constantly think about what information agents learn from the conduct of monetary policy. Central Bankers therefore display constant alertness on how such information changes the behavior of agents which, in its turn, affects the very incentives and the constraints faced by policy-makers. Being the informational content of monetary policy of relevance to the strategic thinking of policy-makers, this thesis has argued, it must also be of irresistible interest to students of macroeconomic theory.

Appendix A

A.1 Analysis of Equation (2.3.45)

We stated in the main text that equation (2.3.45) holds true though this is hard to prove analytically. This is equivalent to stating that the following expression also holds true:

$$\left[E^{tv}\left(\frac{3M_m^0}{P_m}\right)^{\alpha\beta} - E^{tv}\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right] - \left[E^{sv}\left(\frac{3M_m^0}{P_m}\right)^{\alpha\beta} - E^{sv}\left(\sum_{m=1}^{m=3}\left(\frac{M_m^0}{P_m}\right)^{\alpha}\right)^{\beta}\right] \geq 0 \tag{A.1.1}$$

Furthermore, we stated in the main text that the left-hand side of the above expression is rising in α and β. We have verified this numerically and aim to report some computations confirming that this is true in this appendix.

Fixing $\sigma = 1.4$ and $\frac{\bar{\epsilon}}{M} = 0.25$ we start in Table A.1 to study the effect of varying α and β. It will be recalled that $a \geq 1$ because of diminishing returns to scale in labor. Furthermore, recall that $b \geq 1$ since the marginal dis-utility of work is increasing in the amount of labor supplied.

Table (A.1) illustrates that, for this constellation of variables, expression (2.3.45) is non-negative.

The minimum value attained is zero, which is registered when $\alpha = \beta = 1$. This has a pretty intuitive explanation: When $\alpha = \beta = 1$, there is no incentive to smooth out labor across different states of the world as diminishing returns to work do not apply and the dis-utility of labor is linear. Therefore, the fact that demand is more volatile under Transparency Voting (which implies that labor is also going to be more volatile under Transparency Voting) does not create any incentive for agents to locate widely and

$\frac{\bar{\epsilon}}{M} = 0.25, \sigma = 1.4$

	β	1	1.1	1.2	1.3	1.4
α						
1		0	0.006	0.015	0.024	0.034
1.1		0.004	0.024	0.055	0.103	0.174
1.2		0.018	0.058	0.124	0.229	0.391
1.3		0.043	0.113	0.228	0.412	0.70
1.4		0.083	0.192	0.373	0.664	1.12

Table A.1: Value of eq. ((2.3.45): Simulation Results Varying Parameters α and β

$\alpha = \beta = 1.2$

	σ	1.1	1.2	1.3	1.4
$\frac{\bar{\epsilon}}{M}$					
$\frac{1}{4}$		0.226	0.164	0.139	0.124
$\frac{1}{5}$		0.183	0.134	0.113	0.101
		0.157	0.115	0.097	0.086
$\frac{1}{6}$		0.157	0.115	0.097	0.086
$\frac{1}{7}$		0.139	0.101	0.086	0.077

Table A.2: Value of eq. (2.3.45): Simulation Results Varying Parameters σ and $\frac{\bar{\epsilon}}{M}$

insure against macroeconomic risk if $\alpha = \beta = 1$.

Table A.1 shows that whenever $\alpha > 1$ and $\beta > 1$, the expression is always positive and increasing in both α and β.

Why is the expression increasing in α? The answer lies in the fact that the more labor in each island runs into diminishing marginal returns the more agents will have an incentive to smooth out labor across states of the world by locating in all islands hence diminishing macroeconomic risk. For a similar reason, the incentive to locate widely, once τ is taken as a sunk cost, is also rising in β.

An important caveat must be formulated. The numerical values produced in the table do not have any *cardinal interpretations*, but only an ordinal one. In fact, we are comparing across various states of the world and regimes the value attained by an expected utility function and utility theory is usually interpreted as having a cardinal interpretation.

We have experimented with many possible constellations of values for the other parameters and we have always found that the expression is always non-negative; it is rising in both α and β; it is equal to zero with $\alpha = \beta = 1$ and positive otherwise.

We have then proceeded to study the impact of the ratio $\frac{\bar{\epsilon}}{M}$ and of σ on the expression of (A.1.1). We illustrate the result of one of our trials in Table A.2. We now fix α and β and vary the two parameters whose impact we are studying.

The results of Table A.1 confirm that the incentive to locate widely is rising in $\frac{\bar{\epsilon}}{M}$, a result for which we now give an intuitive account. The greater are the fluctuations in monetary aggregates the Central Bank has to stabilize, the greater is the volatility in aggregate demand and employment induced by Transparency Voting and hence the more Transparency Voting gives agents an incentive to hedge against macroeconomic risk by locating their economic activities widely. Hence Transparent Voting makes it more likely that firms locate their productive activities widely across all regions of the Monetary Union when $\frac{\bar{\epsilon}}{M}$ is high.

An increase in σ rises market-power held by firms by making goods less substitutable for consumers, and hence increasing the degree of mark-up of prices over cost. This tends to lower the equilibrium level of output, so that the problem of diminishing returns to labor becomes less pressing when σ is high. For this reason the volatilty in employment induced by Secret Voting imposes a smaller dis-utility to agents when σ is high, which accounts for the result of Table (A.2) which shows that the incentive to locate widely is diminishing in σ.

We found in all the trials we conducted the expression to be rising in $\frac{\bar{\epsilon}}{M}$ and to be diminishing in σ.

A.2 An Extension of the Model when the Inflation Rate Differs among Member Countries

The aim of this appendix lies in arguing that the results of Section 2.2 generalize to a more complicated framework in which the rate of inflation is not constant across the member countries of a Monetary Union.

Such assumption is often made for simplicity and justified by claiming that, if inflation is held to be the instrument of the common monetary policy in a stylized model of a Monetary Union, then it is normal to assume that inflation shall be constant across member countries.

Alternatively, one can sustain that the inflation rate would be constant across member countries of the Monetary Union by arguing that Purchasing Power Parity imposes such restriction. Such simplification is often adopted when analyzing output inflation trade-offs in a Monetary Union, as for instance in Dixit et al. (Dixit and L.Lambertini 2000), Monticelli (Monticelli 2000), Krugman (Krugman 1995) and Pagano (Giavazzi and Pagano 1988).

The loss function in each country, as in the main body of the chapter, is quadratic in the deviation of output from its average level and in inflation:

$$L_i = \left(y - \hat{y}\right)^2 + \beta\left(\pi_i\right)^2; \tag{A.2.1}$$

The Philipps curve has the same structure as posited in the main body of the chapter:

$$y_i = \hat{y} + \gamma\left(\pi - \pi^e\right) + z_i; \tag{A.2.2}$$

Where π^e is the expected rate of inflation, and z_i is shock to output which takes the same form as in the main body of the chapter.

Inflation, which differs across countries in spite of a common monetary policy, consists of two components:

$$\pi_i = \Delta m_i + \mu\Big(y_i - \hat{y}\Big); \tag{A.2.3}$$

The first term captures the component of inflation which is controlled by Monetary Policy, which we take to be the change in the money supply. The second captures the component of inflation which, instead, is driven by the output gap in each country.

Under independent monetary policy each country is free to set the instrument Δm_i as it likes. Instead, under a Monetary Union a common monetary policy implies that Δm is the same across countries.

We follow the same strategy as in the main text. We first derive the optimal conduct of Monetary Policy under independence and then analyze the behavior of the Central Bank of a Monetary Union under both Transparency Voting and Secret Voting.

The Central Bank controls the change in the money supply, while agents are endowed with rational expectations. Given the above assumptions, agents rationally expect $\pi^e = 0$. In fact, given that $E(z_i) = 0$, $E(\Delta m_i = 0)$.

We substitute (A.2.2) into (A.2.3) and solve for π_i to obtain the rate of inflation prevailing in each country under independent monetary policy as a function of Δm_i.

$$\pi_i = \frac{\Delta m_i + \mu z_i}{1 - \gamma\mu}; \tag{A.2.4}$$

Ploughing this back into (A.2.2) and into the loss function of (A.2.1), we verify that the Monetary Authority chooses Δm_i to minimize:

$$L_i = \left(\gamma\left(\frac{\Delta m_i + \mu z_i}{1 - \gamma\mu}\right) + z_i\right)^2 + \left(\frac{\Delta m_i + \mu z_i}{1 - \gamma\mu}\right)^2; \tag{A.2.5}$$

Hence the optimal rate for Monetery Policy set by each country under independence:

$$\Delta m_i^* = -z_i\Big(1 + \mu(\gamma^2 - \gamma + 1)\Big); \tag{A.2.6}$$

Note that monetary policy always leans in the opposite direction to output supply shocks as $\Big(1 + \mu(\gamma^2 - \gamma + 1)\Big) > 0$.

Ploughing (A.2.6) into (A.2.4) the prevailing inflation rate in each country, after that the Monetary Authority has set monetary policy in an optimal manner, turns out to be:

$$\pi_i^* = \frac{-z_i\Big(1+\mu(\gamma^2-\gamma)\Big)}{1-\mu\gamma}; \frac{1}{1-\gamma} < \mu\gamma < 1; \tag{A.2.7}$$

The restriction that $\frac{1}{1-\gamma} < \mu\gamma < 1$ is imposed to ensure that a positive (negative) output supply shock does not, counter-factually, lead to deflation (inflation).

Using an analogous argument as we did in the main text of the chapter, Voting Secrecy in a Monetary Union implies that monetary policy is set according to:

$$\Delta m^{sv,*} = -\overline{z}\Big(1+\mu(\gamma^2-\gamma+1)\Big); \tag{A.2.8}$$

Ploughing (A.2.8) into (A.2.4), we verify that inflation still differs across member countries under Secret Voting and it is equal to:

$$\pi_i^{sv} = \frac{-\overline{z}\Big(1+\mu(\gamma^2-\gamma+1)\Big)+\mu z_i}{1-\mu\gamma}; \tag{A.2.9}$$

Under Transparency Voting, instead, the rate of change in the money supply, denoting again with z^{mv} the shock occurring to the median voter, is dictated by the median voter and equal to:

$$\Delta m^{tv,*} = -z^{mv}\Big(1+\mu(\gamma^2-\gamma+1)\Big); \tag{A.2.10}$$

Substituting (A.2.10) into (A.2.4) yields the inflation rate in each country under Voting Transparency:

$$\pi_i^{tv} = \frac{-z^{mv}\Big(1+\mu(\gamma^2-\gamma+1)\Big)+\mu z_i}{1-\mu\gamma}; \tag{A.2.11}$$

We now show that the results presented in Section 2.2 are robust to the extension presented in this appendix. A set of important observations are in order.

Note that:

$$E\Big[\big(\Delta m^{tv,*}-\Delta m^{sv,*}\big)^2\Big] = E\Big[z^{mv} - \frac{z_+ z_c + z_w}{3}\Big]^2 2\big(1+\mu(\gamma^2-\gamma+1)\big)^2; \tag{A.2.12}$$

Recalling that the analogous expression for the expected square difference of the instrument variable of (2.2.37) across regimes in the model presented in the main text was shown to be:

$$\left(\pi^{*,tv}-\pi^{sv,*}\right)=-(\beta+\gamma)^{-1}\left[\frac{z^{e}+z^{c}+z^{w}}{3}-z^{mv}\right]; \tag{A.2.13}$$

We can see that (A.2.12) and (2.2.37) share some properties. In particular, they are both directly proportional to $\left[\frac{z^{e}+z^{c}+z^{w}}{3}-z^{mv}\right]$. Hence, if one is rising (diminishing) in parameter M, also the other one is rising (diminishing) in parameter M or D.

Furthermore, note that:

$$E\left[\left(\Delta m^{c,*}-\Delta m^{tv,*}\right)^{2}\right]=E\left[z^{mv}-z_{c}\right]^{2}2\left(1+\mu(\gamma^{2}-\gamma+1)\right)^{2}; \tag{A.2.14}$$

And observe also that:

$$E\left[\left(\Delta m^{c,*}-\Delta m^{sv,*}\right)^{2}\right]=E\left[\overline{z}-z_{c}\right]^{2}2\left(1+\mu(\gamma^{2}-\gamma+1)\right)^{2}; \tag{A.2.15}$$

Recalling that equation (2.2.46) states that:

$$E\left(\pi^{*c}-\pi^{tv,*}\right)^{2}=E\left[\frac{z^{mv}-z^{*c}}{\beta+\gamma}\right]^{2}; \tag{A.2.16}$$

We can see that (A.2.14) and (2.2.46) also display an important similarity. Whenever (A.2.14) is increasing (decreasing) in M or D, also (2.2.46) is increasing (decreasing) in M or D. The same remark also holds when we compare (A.2.15) with (2.2.48).

Notice also that:

$$\frac{\partial L_{e,c,w}(\Delta m=\Delta m^{sv,*})}{\partial \Delta m}=0; \tag{A.2.17}$$

This is true as, by definition, Secrecy Voting minimizes the Union wide loss function.

Furthermore, note also that given that $\Delta m^{c,*}$ minimizes the loss function for the Center:

$$\frac{\partial L_{c}(\Delta m=\Delta m^{c,*})}{\partial \Delta m}=0; \tag{A.2.18}$$

Finally, notice also that:

$$\frac{(\partial^2 L)}{(\partial \Delta m)^2} = \frac{2\gamma + 1}{1 - \gamma\mu} > 0; \tag{A.2.19}$$

To sign this expression, we exploited the restriction on γ μ imposed above.

Given this set of observations, we show the setup of this appendix would deliver very similar results to the ones presented in the main text.

In fact, to prove the various propositions stated in the main body we would be carrying out the same set of Taylor approximations as we did in the main body of the chapter, with the only difference that all terms in (π) would be replaced with one of the corresponding expressions above in $(\Delta m.)$ However, we have shown above that all the critical terms in this set of Taylor approximations have the same properties in the model without a common inflationary rate proposed in this appendix as they do in the version formulated in the main text.

Thus the results presented in the main body of the chapter are robust to the extension put forward in the appendix in which the inflation rate differs across member states of the Monetary Union.

Appendix B

B.1 Derivations of Simulation Results for the Signaling Game

B.1.0.1 Simulation One

It is a strictly dominant strategy for type $\epsilon_t = 0$ to keep rates on hold and play $\epsilon_j = 0$ under any symmetric outcome of the game. This follows from the assumption of perfect competition (setting k=1). If no shock hits the economy, output can be kept at its first best level without impacting upon the price level- which is also a first best result. The loss function then reaches its global minimum of zero when no interest rate change takes place.

Table (B.1) (where the relevant values for the loss function are depicted) shows that it is also optimal for the type $\epsilon_t = 1$ - experiencing a small output shock - to pool to $\epsilon_j = 0$ and leave rates unchanged. In fact equation (3.3.9) shows that the loss function will always take value ϵ_t^2 whenever interest rates are not changed by a given type in a symmetric equilibrium.

The most favorable alternative outcome type $\epsilon_t = 1$ could ever get lies in separating and playing $\epsilon_j = 1$ attracting beliefs that $E\left[\epsilon_t \middle| \epsilon_j = 1\right] = 1$. Table B.1 shows precisely that the best possible alternative outcome that ϵ_t could get is 1.48. Therefore also type $\epsilon_t = 1$, and by an analogous symmetrical reasoning $\epsilon_j = -1$, decide to pool to no interest rate change.

Notice that the symmetry of the problem plays a double duty role. On one hand, it allows us to predict that, on average, $\Delta r = 0$. On the other, it effectively acts to imply

Relevant Values for the Loss Function

ϵ_t	ϵ_j	**Beliefs** $E[\epsilon_t \mid \epsilon_j]$			
1		1	1.5	2	2.5
	1	1.48			
2		2	2.5	3	3.5
	1				
	2	5.36			
3		3	3.5	4	-
	1				
	2				
	3	11.22			
4		-	4	4.5	-
	1				
	2				
	3				
	4	19.14			
5		-	-	-	5
	1				
	2				
	3				
	4				
	5				28.99

Table B.1: Payoff Matrix when $\sigma = 0.8; \phi = \psi = k = 1; a2 = 0.8$

that whenever the Bank decides upon a policy upon observing outcome $\hat{\epsilon}_t$, it is bound to follow a symmetric monetary policy when it observes a shock $-\hat{\epsilon}_t$ of the same magnitude in the opposite regime. This implies that whenever the Central Bank decides to pool to no monetary policy change $\epsilon_j = 0$ for a given type it can act on the knowledge that agents will always expect positive and negative shocks to average each other out, provided that interest rates are on hold, because of the symmetry of the model. Therefore, keeping rates on hold acts to neutralize the pro-cyclical impact of wealth effects.

By analogous reasoning, Table B.1 shows, also types $\epsilon_t = 2, -2$ and $\epsilon_t = 3, -3$ decide to play $\epsilon_t = 0$ and keep rates on hold. Type $\epsilon_t = 2$ could, at best, face a loss function of 5.36 when hiking rates by declaring $\epsilon_j = 2$ and facing wealth effects of magnitude $E\left[\epsilon_t \middle| \epsilon_j = 2\right] = 2$. It can always do better by pooling to $\epsilon_j = 0$ and achieve a score of four points in the loss function. Again the property of symmetry ensures that wealth effects will be neutralized at $\epsilon_j = 0$; also type $\epsilon_t = -2$ will keep rates on hold and consumers should expect the shock ϵ_t to be zero.

Types $\epsilon_t = 3, -3$ will also decide to keep rates on hold in spite of the relatively large shock that has hit the economy. It is precisely the considerable size of the shock to aggregate demand that provides the Central Bank with a strong incentive to conceal from the representative agents information on wealth effects. If a separating equilibrium was to be played, Table B.1 shows that the loss function would, at best, take a value of 11.12 (which is greater than $\epsilon_t^2 = 4$ to be achieved by playing a pooling strategy), with rates been hiked by playing strategy $\epsilon_j = 3$; types $\epsilon_t = 4, -4$ and $\epsilon_t = 5, -5$ will also have no incentive to deviate from playing $\epsilon_j = 0$ and getting a payoff of ϵ_t^2.

The Central Bank will not change rates even when it has observed shocks to output of magnitude $\epsilon_t = -4, +4$ or $\epsilon_t = 5, -5$. In fact, Table B.1 shows that wealth effects from deviating from the pooling equilibrium and playing $\epsilon_t = 4, -4$ and $\epsilon_j = 5, -5$ attracting beliefs $E\left[\epsilon_t \middle| \epsilon_j = 5\right] = \epsilon_t$ amounts respectively to 19.14 and 28.99.

The informational content of interest rates generates wealth effects in the analyzed scenario that dominate over the investment effect of interest rates. It is always too costly for the Central Bank to reveal the sign and the magnitude of the shock it has observed and change interest rates.

B.1.0.2 Simulation Two

Values for the Loss Function

ϵ_t	ϵ_j	Beliefs $E[\epsilon_t \vert \epsilon_j]$			
1		1	1.5	2	2.5
	1	1.17	1.50	1.84	2.21
2		1.5	2	2.5	3
	1	4.31	4.87	5.44	6.02
	2	3.81	4.37	4.94	5.52
3		2.5	3	3.5	4
	1	10.41	11.14	11.87	12.61
	2	8.91	9.64	10.37	11.10
	3	8.41	9.14	9.87	10.61
4		3.5	4	4.5	-
	1	19.31	20.21	21.12	
	2	16.81	17.71	18.62	
	3	15.31	16.21	17.12	
	4	14.81	15.71	16.62	
5		-	4	4.5	5
	1		29.82	30.88	31.94
	2		26.32	27.38	28.44
	3	23.82	24.88	25.94	
	4	22.32	23.38	24.44	
	5	21.82	22.88	23.94	

Table B.2: Payoff Matrix when $\sigma = 0.53; \phi = \psi = k = 1; a2 = 0.8$

The Central Bank is always better off pooling to $\epsilon_t = 0$ and facing a loss of ϵ_t^2 rather than revealing its type whenever $-4 < \epsilon_t < 4$, just as in the previous case. This can be seen by inspecting Table B.2.

Type $\epsilon_t = 4$ would be tempted to set interest rates as if it were not facing asymmetric information and set $\epsilon_j = 4$ achieving a loss function of value 15.71. However, if it does so the Table B.2 shows that type $\epsilon_t = 5$ would also set $\epsilon_j = 4$ and therefore type $\epsilon_t = 4$ would face beliefs that the economy has over-heated by a greater extent than what it actually did. At $\epsilon_j = 4$ beliefs are set at $E\left[\epsilon_t \middle| \epsilon_j = 4\right] = 4.5$ and type $\epsilon_t = 4$ faces a payoff of 16.62.

Type $\epsilon_t = 4$ decides to engage into a monetary policy equivalent of the industrial organization concept of limit pricing. It decides to change interest rates by an amount which is just sufficient to differentiate itself from the type that has observed $\epsilon_t = 5$. This is accomplished by hiking rates by an amount ϵ_j^* that makes $\epsilon_t = 5$ just indifferent between pooling as if it were $\epsilon_t = 4$ (by playing $\epsilon_j = \epsilon_j^*$) or separating by playing $\epsilon_t = 5$ so that:

$$\mathbf{L}\left[\epsilon_t = 5, \epsilon_j = 5, E\left[\epsilon_t \middle| \epsilon_j\right] = 5, \phi = \psi = k = 1, a2 = 0.8\right] = \\ = \mathbf{L}\left[\epsilon_t = 5, \epsilon_j = \epsilon_j^*, E\left[\epsilon_t \middle| \epsilon_j\right] = 4.5, \phi = \psi = k = 1, a2 = 0.8\right] \tag{B.1.1}$$

The equation is solved by letting $\epsilon_j* = 3.54$. For values of ϵ_j equal or slightly below ϵ_j*, the representative consumer must believe that type $\epsilon_t = 4$ by iterating the Cho-Kreps refinement criterion. In fact, the criterion rules out beliefs which involve a given type getting with certainty a payoff below its equilibrium one.

It remains to be checked that type $\epsilon_t = 4$ would prefer engaging into limit pricing rather than pooling. This is indeed the case, because by playing the limit pricing strategy it gets $L\left[\epsilon_t = 4, \epsilon_j = 3.54, E\left[\epsilon_t \middle| \epsilon_j\right] = 4, -\right] = 15.82$ which yields a more favorable outcome than pooling to the zero type.

There is a multiplicity of values that can be assigned to beliefs for $\epsilon_j < 3.54$. However, it is crucial to bear in mind that for interest rates hikes involving strategies with ϵ_j greater than $\epsilon_t = 3.54$ and smaller then $\epsilon_t = 4$ beliefs must be set at $E\left[\epsilon_t \middle| \epsilon_j\right] = 4.5$. And conversely beliefs off path must drop to $E\left[\epsilon_t \middle| \epsilon_j\right] = 4$ if ϵ_j is smaller than 3.54 but still larger than three. In fact, if type $\epsilon_t = 4$ sets rates above the threshold value of ϵ_j^*, it cannot be ruled out that the economy has been hit by a shock of magnitude five.

B.1.0.3 Simulation Three

Table 3.3 reports the outcome of the non-cooperative signaling game when $\sigma = 0.4$. We have now generated optimal rules for monetary policy which are quite similar to what we would observe if information were symmetric and hence $\epsilon_t = \epsilon_j$ $\forall t$. It can be seen that whenever a shock hits the economy interest rates move pro-cyclically to reduce output fluctuations. Moreover, $\epsilon_j =$ is quite close to ϵ_t for all types.

And yet, even if the monetary game involves perfect separation, ϵ_t still differs from ϵ_j. It is again due to limit pricing that the informational effect of interest rates has an impact on monetary policy in spite of perfect signal separation.

Type $\epsilon_t = 4$ prevents type $\epsilon_t = 5$ from pooling by playing $\epsilon_j = 4$ and worsen the wealth effect type $\epsilon_t = 4$ faces by playing ϵ_j^{4lim} and ensuring that:

Values for the Loss Function

ϵ_t	ϵ_j	Beliefs $E[\epsilon_t\|\epsilon_j]$			
1		1	1.5	2	2.5
	1	0.98	1.20	1.43	1.67
2		2	2.5	3	3.5
	1	4.13	4.51	4.89	5.27
	2	3.63	4.01	4.39	4.77
3		2.5	3	3.5	4
	1	9.34	9.85	10.36	10.87
	2	7.84	8.35	8.86	9.37
	3	-	7.85	8.36	8.87
4		3.5	4	4.5	-
	1	17.45	18.08	18.71	
	2	14.95	15.58	16.21	
	3	13.45	14.08	14.71	
	4	12.95	13.58	14.21	
5		-	4	4.5	5
	1		27.29	28.04	28.79
	2		23.79	24.54	25.29
	3		21.29	22.04	22.79
	4		19.29	20.04	20.79
	5		18.89	19.76.	20.13

Table B.3: Payoff Matrix when $\sigma = 0.4; \phi = \psi = k = 1; a2 = 0.8$

$$\begin{aligned}&\mathbf{L}\left[\epsilon_t=5,\epsilon_j=5,E\left[\epsilon_t\middle|\epsilon_j\right]=5,\sigma=0.4,\phi=\psi=k=1,a2=0.8\right]=\\&=\mathbf{L}\left[\epsilon_t=5,\epsilon_j=\epsilon_j^{4lim},E\left[\epsilon_t\middle|\epsilon_j\right]=4.5,\sigma=0.4,\phi=\psi=k=1,a2=0.8\right];\end{aligned}\tag{B.1.2}$$

The separation is achieved by setting ϵ_j^{4lim} equal to 3.78. In so doing $\epsilon_t = 4$ achieves a payoff of 13.60 (which is smaller that ϵ_t^2) and therefore prefers to separate via limit pricing rather than pool to the zero shock type by leaving rates unchanged.

However, Table 3.3 shows, type $\epsilon_t = 4$ could achieve a even lower dis-utility of 13.45 by pooling as if it were type $\epsilon_t = 3$ and playing $\epsilon_j = 3$, facing beliefs of $E\left[\epsilon_t\middle|\epsilon_j = 3\right] = 3.5$ which soften the magnitude of pro-cylical wealth effects. Therefore type $\epsilon_t = 3$ must prevent this from happening by ensuring that ϵ_j^{3lim} makes type $\epsilon_t = 4$ indifferent as to pool to type three or stick to its separating strategy:

$$\begin{aligned}&\mathbf{L}\left[\epsilon_t=4,\epsilon_j=3.78,E\left[\epsilon_t\middle|\epsilon_j\right]=4,\sigma=0.4,\phi=\psi=k=1,a2=0.8\right]=\\&=\mathbf{L}\left[\epsilon_t=4,\epsilon_j=\epsilon_j^{3lim},\sigma=0.4,E\left[\epsilon_t\middle|\epsilon_j\right]=3.5,\phi=\psi=k=1,a2=0.8\right]\end{aligned}\tag{B.1.3}$$

Separation is achieved by playing $\epsilon_j^{3lim} = 2.82$, which implies a loss function of value 7.87. Therefore also type $\epsilon_t = 3$ will be tempted to pool to type $\epsilon_t = 2$ and get dis-utility of 7.84 facing beliefs $E\left[\epsilon_t\middle|\epsilon_j = 3\right] = 2.5$ which again act to diminish the amount of pro-cyclical upwards revision on the optimal consumption plan carried out by the representative agent.

Type $\epsilon_t = 2$ therefore also engages into limit pricing to prevent type $\epsilon_t = 3$ from pooling. Type $\epsilon_t = 2$ sets ϵ_j^{2lim} to 1.97 by the same mechanism as in equation (B.1.3) and achieves a payoff of 3.65. Iteration of the Cho-Kreps refinement criterion implies that it cannot be believed that type $\epsilon_t = 3$ plays $\epsilon_j = 1.97$ and get less than its equilibrium payoff even if beliefs were set at $E\left[\epsilon_t\middle|\epsilon_j\right] = 2.5$.

Note that type $\epsilon_t = 1$ does not need to engage into limit pricing. Type $\epsilon_t = 2$ has no incentive to pool as if it were type ϵ_t and play $\epsilon_j = 1$ with beliefs $E\left[\epsilon_t\middle|\epsilon_j = 1\right] = 1.5$, which would get the Central Bank a payoff of 3.75. Type $\epsilon_t = 2$ prefers instead to play $\epsilon_j^{2lim} = 1.97$ and get a loss of 3.62, so that type $\epsilon_t = 1$ does not have to fear to be pooled with an higher type and worsen the wealth effects induced by monetary policy.

B.1.0.4 Simulation Four

Table B.4 illustrates the relevant portion of the payoff matrix for the last simulation we present. Type $\epsilon_t = 5$ has no incentive to try to pool to the same information set as type $\epsilon_t = 4$. In fact, type $\epsilon_t = 5$ could get dis-utility 16.55 by pooling to $\epsilon_j = 4$ and facing beliefs $E\left[\epsilon_t \middle| \epsilon_j = 4\right] = 4.5$. But it can get a lower dis-utility by playing $\epsilon_j = 5$ and achieving 16.38.

Visual inspection of the table confirms that a similar chain of reasoning holds for all types. Type ϵ_t faces a smaller utility function by playing $\epsilon_j = \epsilon_t$ and facing beliefs $E\left[\epsilon_t \middle| \epsilon_j\right] = \epsilon_t$ rather than pooling via strategy $\epsilon_j = \epsilon_t - 1$ facing beliefs $E\left[\epsilon_t \middle| \epsilon_j = \epsilon_t - 1\right] = \epsilon_t - \frac{1}{2}$.

Other deviations could, in principle, make the separating equilibrium unravel. However, the table shows that separating to $\epsilon_j = \epsilon_t$ is a strictly dominant strategy for all types.

The informational content of interest rates has now fallen to a level which is so low that monetary policy triggers off very weak pro-cyclical wealth effects. Only twenty per cent of the population will revise upwards its consumption path after that interest rates have been hiked. The converse holds true if interest rates are lowered, given the symmetry of the model. It therefore follows that the Central Bank will set, if parameters take on this configuration of values, monetary policy with the main purpose of steering investment and money creation by the banking sector in a counter-cyclical fashion.

Values for the Loss Function

ϵ_t	ϵ_j	**Beliefs** $E[\epsilon_t \mid \epsilon_j]$			
1		1	1.5	2	2.5
	1	0.72	0.81	0.90	1
2		2	2.5	3	3.5
	1	3.25	3.41	3.57	3.73
	2	2.75	2.91	3.07	3.28
3		2.5	3	3.5	4
	1	7.83	8.06	8.28	8.50
	2	6.33	6.56	6.78	7
	3	5.83	6.06	6.28	6.50
4		3.5	4	4.5	
	1	14.82	15.10	15.38	
	2	12.32	12.60	12.88	
	3	10.82	11.10	11.38	
	4	10.32	10.60	10.88	
5		4	4.5	5	
	1	23.71	24.05	24.38	
	2	20.21	20.55	20.88	
	3	17.71	18.05	18.38	
	4	16.21	16.55	16.88	
	5	15.71	16.05	16.38	

Table B.4: Payoff Matrix when $\sigma = 0.2; \phi = \psi = k = 1; a2 = 0.8$

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